Horizontal Mergers in Industries with Congestion Effects and Capacity-Enhancing Investments

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Abstract

This paper presents a model for analysing the effects of mergers in industries with price competition, capacity-enhancing investments, and in the presence of congested capacitated networks. It advances beyond traditional models with cost-reducing or quality-enhancing investments by integrating a capacity-sharing approach into a representative consumer framework. The paper compares the quality and capacity investments models in terms of pricing, investment intensities, and merger effects, and discusses how to calibrate the capacity model with real-world data to quantify merger effects.

1. Introduction

This paper introduces a model of price and capacity investment competition that can be calibrated to quantify the effects of mergers on prices, quantities, investment, congestion, consumer surplus, and overall welfare. Despite the intense debate among practitioners and competition authorities, there is limited research on the impact of horizontal mergers in scenarios where firms compete by setting prices and investment levels (Motta and Tarantino, 2021). This gap is particularly relevant in industries like telecommunications, where recent years have seen a severe increase in traffic volume, necessitating substantial investment in infrastructure and network capacity by operators.

The few existing papers that explore horizontal mergers in the presence of price and investment competition (e.g., Federico et al., 2018; Motta and Tarantino, 2021; Bourreau, Jullien, and Lefouili, 2021) typically model investments as either impacting marginal cost (cost-reducing investments or process innovation) or shifting the demand function (quality-enhancing investments or product innovation). As discussed below, such models may not capture some key characteristics of industries such as telecommunications where network

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capacity is crucial to maintaining service quality. Congested capacitated models (or capacity models for short) are particularly suitable for these industries. These models generally apply to communication networks, including transportation and electricity industries, and have proven especially useful in industrial organization for examining the effects of net neutrality regulation (see, for instance, Choi and Kim, 2010; Bourreau, Kourandi, and Valletti, 2015) in environments where network traffic can lead to congestion, affecting both content providers and end-users. These papers explore the impact of net neutrality regulation on capacity investments in the internet access market and on content market innovation. Additionally, capacity models are key to understanding potential inefficiencies arising in the expansion of modern communication networks (Acemoglu, Bimpikis, and Ozdaglar, 2009). Congestion effects under capacity constraints have also been explored in oligopolistic competition models in general frameworks and applied to the telecommunications and transport sectors (Xiao, Yang and Han, 2007; Acemoglu, Bimpikis, and Ozdaglar, 2009), and on other topics such as market entry (Johari, Weintraub, and Van Roy, 2010).

This paper sets out to achieve two primary objectives: firstly, to compare the capacity model with a standard model of quality-enhancing investments (or ‘quality model’ for short). This comparison aims to deepen our understanding of the distinctions between the two models in terms of pricing, investment intensity and merger effects. Secondly, we aim to develop a straightforward methodology for calibrating the model for practical application, particularly in telecommunications markets where capacity constraints, traffic congestion, and investments play a crucial role. This calibrated model will enable practitioners and competition authorities to quantify the effects of a horizontal merger between two firms in the presence of congestion costs, while considering any efficiencies.

In a capacity model, the cost of traffic congestion experienced by users can be quantified in monetary terms by calculating the average level of delay or waiting time for given traffic flows and network capacity, and then determining its cost to the user. In this paper, we show how incorporating the capacity-sharing model into a representative consumer model yields inverse demand functions that are linear in quantities. Motta and Tarantino (2021) show that known effects of a merger with cost-reducing investments extend to models with demand-enhancing investments under two specific types of demand, making the demand-enhancing investment isomorphic to cost-reducing investment. Specifically, this occurs in quality-adjusted models and models showing a hedonic price transformation. Neither of these cases seems feasible in the capacity-sharing model, wherein the level of investment interacts with output within the inverse demand function by appearing in the denominator of the slope. This formulation justifies a fresh analysis of merger effects in this type of model.

In the capacity-sharing model investments rotate the inverse demand function (i.e. consumers' willingness to pay does not increase at the origin), unlike in popular models with quality-enhancing investments where investments typically shift the inverse demand function outwards. We discuss how this distinction positions the capacity-sharing model as an adequate representation of industries facing capacity constraints and congestion issues. Our examination of investment and price competition uncovers notable differences between the two models. While in the quality model, the firm's increase prices in response to industry-wide investment increases, in the capacity model, firms *reduce* their prices in response. This pricing strategy is partially driven by the firm's increased ability to capture new demand when lowering its prices, compared to before increasing its investment.

Investment intensity. Although the model does not yield explicit solutions, our numerical analysis reveals that with increasing product substitutability and thus fiercer price competition, both models exhibit a decline in profits. However, the investment strategies and intensity diverge significantly in the presence or absence of capacity constraints. In the quality model, investment intensity (defined as investment cost over total revenue) is highly variable and can surge to 100% of revenues when products are closely substitutable, and investment has a substantial impact on consumer utility. This contrasts with the capacity-sharing model, when investments significantly reduce congestion costs, investment intensity remains consistently high across various degrees of product substitutability, and, moreover, never reaches the peaks observed in the quality model – in our examples, it never exceeds 50% of revenues.[2] This contrast is due to the shape of investments in the two models: in the quality model investment is U-shaped with respect to the degree of product substitutability, whereas in the capacity model investment decreases monotonically.

Efficiency analysis. We also explore how efficiency gains in investments, similar to those from a merger, affect investments and prices in both models. In the quality model, a 50%

[2] The investment intensity (capital expenditure over total revenue) for telecom companies often reaches around 20 — 25% (Moss Adams, 2022, '2022 Telecommunications Benchmarking Study'). The average investment intensity of EU-15 countries from 2005 to 2007 (as a percentage of value added) is measured at 22.3% for telecommunications sector. Other industries with high investment intensity include energy (37.8%), water, waste (42.6%), air transport (42.1%), and warehousing (42.2%), with water transport being the only sector above 50% (56.3%) (European Commission, 2022, 'Science, Research and Innovation Performance of the EU 2022 – Building a Sustainable Future in Uncertain Times', Table 9-1, p. 612).

improvement in investment cost efficiency significantly boosts investment by over 100%, raising equilibrium prices. However, despite this price increase, consumers benefit from the higher levels of investment via an improvement in quality, leading to a significant expansion in demand. The capacity-sharing model shows a 30% increase in investment, with prices *decreasing* as investment levels rise. As a result, consumers also benefit from lower prices and lower congestion, which also leads to an increase in demand.

Merger without efficiencies. Absent synergies, the quality model indicates that mergers lead to substantial increases in investment for the non-merging firms, especially in scenarios of high investment intensity. In contrast, the merging firms notably reduce their investment. Post-merger, all firms set higher prices. With low investment intensity, the merging firms exhibit more substantial price increases than the non-merging firms, but this pattern reverses in environments of high investment intensity when products are moderately to highly substitutable. The capacity-sharing model, however, predicts a more moderated response in investments. Non-merging firms moderately increase their investment levels, while merging firms typically reduce theirs. Unlike in the quality model, the price increases for merging firms exceed those of non-merging firms irrespective of the level of investment intensity. Despite these differences, the overall impact on consumer surplus remains relatively close across both models, though in most cases it is slightly weaker in the capacity model. The difference becomes more apparent in scenarios of moderate to high product differentiation combined with high investment intensity, where the reduction in consumer surplus in the capacity model is half as much as that observed in the quality model.

Merger with efficiencies. When synergies that reduce investment costs come into play, thereby elevating investment levels, merger effects are more pronounced in the quality model, resulting in notable rises in both prices and investments. We observe a consistent increase in total surplus in both models in the presence of high investment intensity. The quality model also predicts an increase in consumer surplus, whereas the capacity model predicts an increase in consumer surplus for moderately to highly differentiated products. This suggests that merger efficiencies, like reduced investment costs, provide greater consumer benefits in markets characterised by differentiated products and high investment intensity.

Model calibration. In Section 6 we adopt a specific investment cost function and then explain in detail how to calibrate the different model parameters. We present different strategies to overcome data limitations and proceed with model calibration in cases where there is no unique way to calibrate parameters, or the analyst lacks the necessary information due to data accessibility issues.

The paper is structured as follows. Section 2 presents the capacity-sharing model, where firms engage in competition via pricing and capacity-expanding investments. Section 3 conducts a comparative analysis of the two models, discusses their economic interpretations, and analyses pricing decisions and investment intensity. Section 4 offers a thorough examination of a symmetric duopoly and compares the simulation results of pricing and investment decisions between the two models. Section 5 describes the merger effects in the capacity-sharing model, presents various efficiencies typically generated by mergers, and explains how to incorporate them into the model in a tractable manner. Furthermore, this section conducts simulations to compare the effects of mergers across the two models. Section 6 provides a step-by-step guide on how to calibrate the model. Finally, Section 7 concludes the paper.

2. A Model of Capacity and Price Competition

We consider an industry with $n \geq 2$ firms, where each firm $i = 1, \dots, n$ produces a differentiated good or service and operates at a constant, possibly different, marginal cost of $c_i \geq 0$.

We account for congestion effects: the benefits consumers derive from purchasing from or subscribing to a specific firm are diminished by a negative externality that increases monotonically with the total volume of consumption that the firm serves. Each firm i can invest u_i to expand its network capacity (infrastructure, network deployment, number of sites or facilities...) and mitigate these congestion effects. Thus, firms compete in prices and investment levels, with each firm i simultaneously setting its price p_i and capacity-expanding investment level u_i.

Investment costs. The cost of investing u_i is given by the investment cost function: $\Gamma_i(u_i)$, which satisfies $\Gamma_i(0) = 0$, $\Gamma_i{}' > 0$, and $\Gamma_i{}'' \geq 0$. Allowing for $\Gamma'' > 0$ is motivated by both technical and economic considerations.

From a technical standpoint, increasing the convexity of the function ensures that the optimization problem faced by firms is well-behaved, enhancing the likelihood that the profit function will satisfy second-order conditions.[3]

From an economic perspective, there are compelling reasons to justify a strictly convex investment cost function. As a firm's total investment increases, the cost of investing may rise progressively. For instance, when a firm increases its capital expenditure (CAPEX) by investing in new equipment, machinery, or facilities, captured here by u_i, this may additionally result in higher operating expenditures (OPEX) due to ongoing operational costs such as maintenance, utilities, personnel, or insurance. The function $\Gamma_i(u_i)$ accounts for the full cost of investing in and maintaining the network. Additionally, the cost of capital and financial needs might grow more than proportionally as investment scales up.[4] These situations imply that the rate of growth of the investment cost function is strictly increasing: $\Gamma_i'' > 0$.[5]

Additionally, assuming $\Gamma_i'' > 0$ may be necessary when u_i serves as a proxy variable, which can be useful in quantitative analyses. For example, in the telecommunications industry u_i might represent the number of operational sites of firm i, the bandwidth capacity available to its customers, the rollout of new technologies such as 5G networks, or some quantitative measure or indicator of the extent to which a firm has expanded or improved its network infrastructure, such as network coverage, capacity of existing facilities or technologies to enhance service quality and data speeds.

Congestion costs. The congestion cost is typically measured in currency equivalent terms.[6] As a result, the consumer's utility of buying from or subscribing to firm i depends on

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[3] When the parameter t_i, to be introduced later in this paper, is low, congestion costs become highly sensitive to investment. This sensitivity can potentially lead to the non-existence of equilibrium when t_i is sufficiently low and $\Gamma_i = u_i$. However, increasing the convexity of the investment cost function, such as by adopting a quadratic form ${u_i}^2/2$, may ensure the concavity of the profit function and the existence of an equilibrium. For this reason, we assume this functional form in our analysis of duopolistic competition.

[4] Similarly, labour, management, and departmental size, among other factors, face capacity constraints. As these limits are approached, costs increase sharply due to the need for expansion.

[5] Recent papers that make the same assumption include Motta and Tarantino (2021) and López and Vives (2019), with the latter interpreting the investment variable as spending on R&D. Athey and Schmutzler (2001) analyse a general model of oligopolistic competition with investment, which encompasses as special cases many well-known models of competition and investments. They introduce a general investment cost function and require sufficient convexity in this function to ensure the uniqueness of the equilibrium across the various models (Lemma 2, p. 7).

[6] See for example Acemoglu and Ozdaglar (2007); Hayrapetyan, Tardos and Wexler (2007); Johari, Weintraub, and Van Roy (2010); Perakis and Sun (2014); Ozdaglar (2008); Xiao, Yang and Han (2007).

the full price of firm i, which is the sum of the price (p_i) and the congestion cost the consumer experiences per unit of consumption, denoted by $l_i(q_i, u_i)$:

$$p_i + l_i(q_i, u_i), \tag{1}$$

where q_i is the quantity supplied by firm i. The congestion cost per unit of consumption of firm i is increasing in its quantity and decreasing in its investment level: $\partial l_i/\partial q_i \geq 0$ and $\partial l_i/\partial u_i \leq 0$.

The *total congestion cost* experienced by consumers of firm i is:

$$L_i(q_i, u_i) = l_i(q_i, u_i)q_i. \tag{2}$$

To model Bertrand pricing with differentiated products, we consider a demand system obtained from a representative consumer with a taste for variety and quasilinear utility.[7] More specifically, we consider the Singh-Vives subutility function (Singh and Vives, 1984), but extended it to include congestion costs[8]:

$$U = \sum_{i=1}^{n} a_i q_i - \frac{1}{2}\left(\sum_{i=1}^{n} \hat{\beta}_i {q_i}^2 + 2\sum_{i}\sum_{j>i} \rho_{ij} q_i q_j\right) - \sum_{i=1}^{n} l_i(q_i, u_i)q_i, \tag{3}$$

where a_i and $\hat{\beta}_i$ are strictly positive and represent the intercept and slope of the inverse demand function, respectively. The parameter ρ_{ij} is a measure of the degree of substitution between the goods. Goods i and j are substitutes if $\rho_{ij} > 0$, independent if $\rho_{ij} = 0$ and complements if $\rho_{ij} < 0$. Throughout the analysis we consider substitute products, thus $\rho_{ij} > 0$. Notice that product substitutability between i and j intensifies as ρ_{ij} increases.

Capacity-sharing model. There exist various frameworks for modelling network congestion in the realm of capacity modelling.[9] Our objective, however, is to develop a model that aligns with the Singh-Vives utility function, ultimately leading to a linear demand system that can be calibrated. To that end, we consider a capacity-sharing model in which each firm owns a processing facility (for example, the network). Additionally, the firm's

[7] The concept of a representative consumer is in line with other economic models that recognize the diversity in consumer preferences. For instance, discrete choice models, where firms account for unobserved consumer preferences as random variables, mirror this approach by having consumers choose the option that maximizes their utility, similar to the representative consumer model. For an in-depth exploration, see Vives (1999). Additionally, see Anderson et al. (1992) for equivalences between representative consumer models and other approaches, such as characteristics and discrete choice models.

[8] This is also known as the Bowley demand model: Bowley (1924) uses a representative consumer with quadratic utility function to derive linear inverse demand functions.

[9] Commonly adopted in the literature are the M/M/1 queuing system (Choi and Kim, 2010; Bourreau, Kourandi and Valletti, 2015) and the capacity-sharing model (Johari, Weintraub, and Van Roy, 2010).

investment determines the processing capacity of its facility per unit time: if firm i invests u_i, then it has the capacity of processing $\Phi(u_i)$ demand units per unit time, where the function $\Phi(u_i)$ is assumed to be concave.

Therefore, if q_i is the total mass of demand at firm i, and capacity is equally shared among consumption requests, then each request faces a processing delay of $q_i/\Phi(u_i)$ time units. This approach to modelling congestion is also adopted in other papers as well. Xiao, Yang and Han (2007) use this functional form to analyse competition among private toll roads. Berstein, DeCroix and Bora Keskin (2020) employ it to examine competition on two-sided platforms with congestion effects on both demand and supply. Johari, Weintraub and Van Roy (2010) propose this model as a suitable for technological services, including wireless Internet service provision.[10] Nguyen et al. (2011) use similar congestion cost assumptions to study the impact of additional unlicensed spectrum on competition in wireless services and congestion. They use a general function for congestion costs as the sum of a fixed component $T_i \geq 0$ and the product of a function α_C and x, where x is the number of customers and α_C is a function that decreases with C (capacity or bandwidth of spectrum). This function encompasses the one presented in this paper.[11]

We assume the following: $\Phi(u_i) = \theta_i u_i$ with $\theta_i > 0$. As such, we may write:

$$l_i(q_i, u_i) = \mu\left(\frac{q_i}{\Phi(u_i)}\right) = \mu\left(\frac{q_i}{\theta_i u_i}\right) \tag{4}$$

with $\mu > 0$. While the parameter θ_i maps investment levels to processing capacity per unit time, the parameter μ maps processing delays to congestion costs. By inserting the above expression into (3) we obtain

$$\widetilde{U} = \sum_{i=1}^{n} a_i q_i - \frac{1}{2}\left(\sum_{i=1}^{n} \hat{\beta}_i {q_i}^2 + 2\sum_{i}\sum_{j>i} \rho_{ij} q_i q_j\right) - \sum_{i=1}^{n}\left(\frac{q_i}{t_i u_i}\right) q_i,$$

[10] The model is discussed in the online appendix (Example EC.2) and referred to as the capacity-sharing model. Its suitability for telecommunication services lies in its implication of constant returns to investment (Lemma EC.1. in the online appendix) which accommodates loss systems. In these systems, the cost to the user is given by the probability that making a call might result in a dropped call in a finite buffer queueing system. This contrasts with other systems such as M/G/1, which encompasses the M/M/1 system, where the service times follow an exponential distribution. The M/G/1 system accommodates delay systems where the cost to the user corresponds to a delay in a queuing system with an infinite buffer. The authors show that under M/G/1 system there are increasing returns to investment, and thus it is typically efficient for a single firm to serve the entire market (Example EC.4. in online appendix).

[11] In fact, in the example of Theorem 3 (p. 151), they assume $T_i = 0$ and $\alpha_C = 1/C$, which implies a congestion cost equal to x/C, essentially the same congestion cost function that we adopt.

where the factor $t_i = \theta_i/\mu$ is an inverse measure of the efficiency of investment relative to congestion costs. A higher t_i (due to either a higher θ_i or a lower μ) indicates that investments in capacity are less effective in offsetting the cost of experiencing delays. The expression can be simplified as

$$\widetilde{U} = \sum_{i=1}^{n} a_i q_i - \frac{1}{2}\left(\sum_{i=1}^{n} \beta_i {q_i}^2 + 2\sum_{i}\sum_{j>i} \rho_{ij} q_i q_j\right), \tag{5}$$

where

$$\beta_i = \hat{\beta}_i + \frac{1}{\tau_i u_i} > 0$$

with $\tau_i \equiv t_i/2 > 0$. The expression (5) represents an extension of the Singh-Vives *subutility* function, modified to define β_i as a decreasing function of the investment level u_i. This formulation is especially useful because it enables us to construct a *utility* function that is separable and quasi-linear in the *numeraire* good. Such a configuration removes income effects, yields linear demands, allows us to perform partial equilibrium analysis and, importantly, to use the consumer surplus as an appropriate measure of consumers' welfare change.[12]

The representative consumer solves $\max_{\boldsymbol{q}} \; \widetilde{U} - \sum_{i=1}^{n} p_i q_i$. Given that the Hessian of $\widetilde{U}$ is negative definite, implying that $\widetilde{U}$ is a concave function of the *n* differentiated goods, we can derive the inverse demand functions, $p_i = P_i(\boldsymbol{q})$, from consumer's problem's first-order conditions.[13] This results in symmetric cross-price effects, where $\partial P_i/\partial q_j = \partial P_j/\partial q_i$ for $j \neq i$, and a downward-sloping inverse demand curve, $\partial P_i/\partial q_i = -\beta_i < 0$. Additionally, inverting the system of inverse demands yields direct demand functions: $D(\cdot) = \big(D_1(\cdot), \dots, D_n(\cdot)\big)$. These direct demands retain properties of being downward-sloping and exhibiting symmetric cross-price effects.

Investments and demand. It is important to note that $\partial P_i/\partial u_i \partial q_i > 0$; that is, when firm i invests in capacity, its inverse demand function rotates outward while maintaining its position at the origin. Such an improvement in capacity decreases congestion costs and, consequently, expands the market: for a given price (lower than the intercept), the quantity demanded for firm i's product increases.

[12] See Vives (1999, Ch. 3).
[13] Specifically, for $q_i > 0$, the inverse demand for firm i is given by $p_i = \partial\widetilde{U}/\partial q_i$.

3. Comparative Analysis of Investment Models

In this section, we undertake a comparison between the capacity-sharing model and the quality-enhancing investments model. For clarity and ease of comparison, we assume symmetry throughout this analysis. This means the demand system we have discussed is symmetric ($a_i = a$, $\hat{\beta}_i = \beta$, $\tau_i = \tau$, $\rho_{ij} = \rho$), characterized by exchangeable functions $q_i = D(p_i, p_{-i}, u_i, u_{-i})$, for $i = 1, \dots, n$, and the investment cost function satisfies $\Gamma_i(u) = \Gamma(u)$.

A common assumption made in models with quality-enhancing investments is that an increase in firm i's investment boosts demand for its product while diminishing demand for its competitor (Motta and Tarantino, 2021; Bourreau, Jullien and Lefouili, 2021), implying $\partial D_i/\partial u_i \geq 0$ and $\partial D_k/\partial u_i \leq 0$ for $i \neq k$. In the capacity-sharing model, these assumptions are also valid in the symmetric equilibrium when $\beta > \rho$, mirroring the conventional assumption that own effects surpass cross-effects.

Our focus is on the most straightforward case of quality-enhancing investments where investment in quality lifts the intercept of the inverse demand function, thereby broadening the market and pushing demand outward.[14] This scenario is prevalent in vertical product differentiation models with quality competition, where users differ in their tastes or, by analogy, in their incomes (Tirole, 1988). A similar parallel is drawn with persuasive advertising models, where investment in advertising increases consumers' willingness to pay by elevating their reservation values (von der Fehr and Stevik, 1998).[15]

To delineate the differences between these models, we next formalize the capacity-sharing model under Assumption A1, and the quality-enhancing investments model under Assumption A1':

Assumption A1. *In a capacity-sharing model*

$$a_i = a \text{ and } \beta_i = \beta + \frac{1}{\tau u_i}$$

, with a and β as strictly positive constants, for $i = 1, \dots, n$.

Assumption A1'. *In a model with quality-enhancing investments*

[14] Recent studies utilizing this framework include Bayona and López (2018), and Motta and Tarantino (2021, pp. 16-17). See also Vives (2008, pp. 454-455).

[15] In this context, firms would rather avoid advertising as its costs, wasted in competition, yield no equilibrium advantage. Belleflamme and Petiz (2010, pp. 149-153) show that a similar outcome occurs in models where advertising shifts the distribution of consumer tastes in favour of the firm. See Bagwell (2007) for a survey on the literature on economic models on advertising.

$$a_i = a + \alpha(u_i) \textit{ and } \beta_i = \beta$$

, where a and β are strictly positive constants, $\alpha(0) = 0$ and $\alpha' \geq 0$ (implying the willingness to pay increases with the level of investment in the product), for $i = 1, \ldots, n$.

Under Assumption A1' the model admits a hedonic price transformation, allowing the game to pivot to one where firms compete in hedonic prices $h_i = p_i - \alpha(u_i)$ and investment levels u_i. This makes the standard quality model of particular interest since, as Motta and Tarantino (2021) underscore, within this framework, the effects of a merger are akin to those in a model with cost-reducing investments, characterized by $a_i = a$, $\beta_i = \beta$, and $c_i(u_i)$ as firm i's marginal cost as a function of its investment level u_i, with $c' < 0$, $c'' \geq 0$, $c''' \geq 0$ and $c(0) \geq 0$.[16] Motta and Tarantino (2021) also show that more intricate quality-adjusted models, akin to those by Sutton (1996) and Symeonidis (2000, 2003), mirror the quality-enhancing investment model in their equivalence to models with cost-reducing investments. Thus, their insights on merger effects in the context of cost-reducing investments extend to these more elaborated demand function types as well. However, for general demand formulations, results remain ambiguous, necessitating a tailored analysis for different demand structures. For example, models that enable firms to differentiate their products through investments typically observe an increase in investment post-merger (Bourreau, Jullien and Lefouili, 2021).

3.1 Models' interpretations

In both models, inverse demands are written as follows:

$$p_i(\boldsymbol{q}) = a_i - \beta_i q_i - \sum_{j \neq i} \rho q_j.$$

Under A1, $\partial p_i/\partial u_i = -\partial \beta_i/\partial u_i = 1/(\tau_i u_i^2) > 0$, but under A1', $\partial p_i/\partial u_i = \partial a_i/\partial u_i = \alpha'(u_i) > 0$. Hence, all else being equal, an increase in a firm's investment level under A1 rotates the inverse demand function outward, whereas under A1', it shifts the function outward. The capacity-sharing model thus more accurately reflects competition in industries where investments alleviate user congestion costs. When the inverse demand function *rotates* outward (i.e. without a change in the intercept), the initial units' willingness to pay remains largely unaffected by the investment, an intuitive outcome given these units' negligible contribution to network congestion. On the contrary, as consumption escalates,

[16] Direct demands $D(\cdot)$ can be expressed as functions of hedonic prices $h_i(= p_i - \alpha(u_i))$ for $i = 1 \ldots n$, allowing us to substitute p_i with $h_i + \alpha(u_i)$ in the profit function to reformulate it as: $\pi_i(h_i, u_i) = [h_i - (c - \alpha(u_i))]D_i(\cdot) - \Gamma_i(u_i)$, where $\alpha \geq 0$.

so does the incremental willingness to pay for each unit, emphasizing the increased value of reducing congestion. However, with a parallel shift in demand (i.e. with an increase at the intercept) – as seen with quality-enhancing investments – the willingness to pay uniformly rises across all units, regardless of whether they are the initial, less congested units or later ones where the network becomes increasingly congested.

3.2 Investments and Pricing

To grasp how investments influence pricing, let us first examine their effect on the firm's marginal profit from adjusting (increasing or decreasing) its price. With symmetric firms, the profit for firm i is given by $\pi_i = (p_i - c)D_i(p_i, p_{-i}, u_i, u_{-i}) - \Gamma(u_i)$, for $i = 1, \ldots, n$. The marginal profit with respect to price then is:

$$\frac{\partial \pi_i}{\partial p_i} = D_i(p_i, p_{-i}, u_i, u_{-i}) + (p_i - c)\frac{\partial D_i(p_i, p_{-i}, u_i, u_{-i})}{\partial p_i}. \tag{6}$$

This marginal profit from altering the price encompasses two familiar effects: the change in revenue from selling the existing quantity at a new price (revenue effect), and the change in profit from the change in demand (demand effect).

To examine the impact of investments on pricing, assume all firms set the same price and investment level with $\partial \pi_i / \partial p_i \geq 0$ (the inequality is binding at the equilibrium). If firm i raises its investment, in both models, this increases demand for firm i ($\partial D_i / \partial u_i > 0$), thus increasing the revenue effect. As a result, the marginal profit from an increase in price rises, giving the firm the incentive to increase its price and recapture some of the new value created for users. Yet, the models diverge in the demand effect. In the quality model, investment does not affect the slope of the inverse demand function (only the intercept): a higher investment, therefore, does not impact marginal profit via the demand effect. On the other hand, in the capacity-sharing model, investment diminishes the absolute value of the slope of the inverse demand function, hence increasing the absolute value of the slope of the demand function. This indicates that after investing, reducing the price can draw more demand, thanks to reduced congestion costs. Conversely, increasing prices results in a more significant reduction in demand, which in turn reduces marginal profit, lessening the incentives to elevate prices in the capacity-sharing model as opposed to the quality-enhancing investments model after an increase in investment.

Formally, by setting rival strategies at a symmetric profile of prices and investments, $p_j = p$ and $u_j = u, j \neq i$, we may analyse the (interior) symmetric equilibrium prices as a function of firms' investment level: $p(u)$. At equilibrium, the price-related first-order condition for firm i simplifies to

$$\phi_i(p;u) \equiv D_i(p;u) + (p-c)\frac{\partial D_i(p;u)}{\partial p_i} = 0, \tag{7}$$

with $p = p(u)$. Differentiating this expression with respect to u yields:

$$\frac{dp}{du} = \frac{\partial \phi_i(p;u)/\partial u}{-\partial \phi_i(p;u)/\partial p}. \tag{8}$$

Given $\partial \phi_i/\partial p < 0$,[17] the sign of dp/du equals the sign of $\partial \phi_i/\partial u$, thus:

$$sign\left\{\frac{dp}{du}\right\} = sign\left\{\frac{\partial D_i(p;u)}{\partial u} + (p-c)\frac{\partial D_i(p;u)}{\partial u \partial p}\right\}. \tag{9}$$

The first term within the braces of (9) is positive under A1 and A1' in a duopoly scenario. Generally, assuming $\partial D_i(p;u)/\partial u > 0$ echoes the assumption that own effects on demand outstrip cross effects after a uniform price increase across all firms, and is a standard assumption in investment models (Bourreau, Jullien and Lefouili, 2021). The second term in (9) is zero under A1'. Therefore, in the quality-enhancing investment model, $dp/du > 0$, indicating firms' propensity to increase prices with a uniform rise in investment. In contrast, under A1 the second term is negative, and, as demonstrated in the two-firm scenario below, surpasses the first term; the capacity-sharing model thus yields $dp/du < 0$.

3.3 Investment intensity

Let $D \equiv D_i(p,p,u,u)$ be the demand for a firm at the symmetric (interior) equilibrium of the full game. Drawing from the first-order condition regarding price, $\partial \pi_i/\partial p_i = 0$, where marginal profit is given by equation (6), we can express:

$$L = \frac{1}{\eta_{D_p}},$$

where $L \equiv \frac{p-c}{p}$ is the Lerner index, and $\eta_{D_p} = -\frac{\partial D_i(p,p,u,u)}{\partial p_i}\frac{p}{D}$ denotes the absolute value of the price elasticity of the demand. At this equilibrium, the first-order condition for investment is also satisfied:

$$\frac{\partial \pi_i}{\partial u_i} = (p-c)\frac{\partial D_i(p,p,u,u)}{\partial u_i} - \Gamma'(u) = 0. \tag{10}$$

[17] This result is inherent to the model's design. Given linear demand in prices, we can write $\partial \phi_i/\partial p = \partial D_i(p;u)/\partial p + \partial D_i(p;u)/\partial p_i$. Furthermore, $\frac{\partial D_i(p;u)}{\partial p} = \frac{\partial D_i(p;u)}{\partial p_i} + \sum_{j\neq i}\frac{\partial D_i(p;u)}{\partial p_j} < 0$ because own effects on demand surpass the cross effects.

This can be reformulated as:

$$L = \frac{\eta_\Gamma}{\eta_{D_u}}\left(\frac{\Gamma(u)}{pD}\right), \tag{11}$$

where $\eta_{D_u} = \frac{\partial D_i(p,p,u,u)}{\partial u_i}\left(\frac{u}{D}\right)$ is the investment elasticity of demand, and $\eta_\Gamma = \Gamma'(u)\left(\frac{u}{\Gamma(u)}\right)$ is the elasticity of the investment cost function. By aligning the two expressions for L, we obtain:

$$r \equiv \frac{\Gamma(u)}{pD} = \frac{\eta_{D_u}}{\eta_{D_p}}\left(\frac{1}{\eta_\Gamma}\right). \tag{12}$$

The above equation establishes the optimal investment expenditure as a fraction of revenue. At the equilibrium, investment intensity, r, equals the quotient of the investment elasticity of demand over the product of the price elasticity of demand and the elasticity of the investment cost function. When $\eta_\Gamma = 1$ (due to $\Gamma(u) = u$), the investment intensity is solely determined by the ratio of the two demand elasticities.

4 Duopolistic Competition

Next, to further clarify the differences between the two models, we examine a scenario involving two firms under Assumptions A1 and A1'. In line with the linear structure of the processing capacity function of the capacity model, $\Phi_i = \theta_i u_i$, under the quality model we posit that investments enhance consumers' utility via the following quality-enhancing function: $\alpha(u_i) = \tau u_i$ with $\tau > 0$. Additionally, we assume a quadratic investment cost function, $\Gamma(u) = u^2/2$, which applies to both models.[18]

4.1 Quality-Enhancing Investments Model

Under A1', the demand for firm i is defined as:

$$D_i(p_i, p_j, u_i, u_j) = \frac{(a_i - p_i)\beta - (a_j - p_j)\rho}{\beta^2 - \rho^2}, \tag{13}$$

where $\beta - \rho > 0$, and $a_i = a + \tau u_i$. Therefore, $\partial D_i/\partial u_i > 0$, $\partial D_i/\partial u_j < 0$ and $\partial D_i/\partial u_i \partial p_i = 0$.

From the corresponding first-order condition, we may obtain the profit-maximizing price, p_i^r, as firm i's best response to the pair (p_j, u_j) for a given investment level u_i: $p_i^r = f_{p_i}(p_j, u_j; u_i)$, $j \neq i$. We obtain that $\partial p_i^r/\partial u_i = \tau/2 > 0$ and $\partial p_i^r/\partial u_j = -\rho\tau/(2\beta) < 0$. As

[18] We assume second-order conditions hold throughout the analysis. Additionally, in the numerical analyses presented later in this paper, we verify that these second-order conditions are indeed met.

can be intuitively derived from the previous analysis, in this model, firms are incentivized to raise prices with their investment levels and to lower them in response an increase in their rival's investment. Solving the first-order condition for prices at uniform price and investment levels, $p_1 = p_2 = p$ and $u_1 = u_2 = u$, yields:

$$p = \frac{(\beta - \rho)(a + u\tau) + c\beta}{2\beta - \rho}, \tag{14}$$

confirming, as discussed above, that $dp/du > 0$. This outcome arises because investment, under A1', shifts the demand outwards, affecting only the revenue effect and leading firms to increase prices after uniformly raising their investment levels.

The equilibrium price and investment levels are:

$$p_1 = p_2 = p = \frac{(\beta^2 - \rho^2)a + [\beta^2 - (\tau^2 - \rho)\beta]c}{\beta^2 - \rho^2 + \beta(\beta + \rho - \tau^2)} \tag{15}$$

and

$$u_1 = u_2 = u = \frac{(a - c)\tau\beta}{\beta^2 - \rho^2 + \beta(\beta + \rho - \tau^2)}. \tag{16}$$

At equilibrium:

$$\eta_{D_p} = \frac{(\beta^2 - \rho^2)a + (\beta + \rho - \tau^2)\beta c}{(\beta^2 - \rho^2)(a - c)}, \tag{17}$$

$\eta_{D_u} = \beta\tau^2/(\beta^2 - \rho^2)$ and $\eta_\Gamma = 2$. With $c = 0$, $\eta_{D_p} = 1$, and as a result the investment intensity is driven by the demand's investment elasticity, $r = \eta_{D_u}/2$. More generally,

$$r = \frac{\beta\tau^2(a - c)}{2[(\beta^2 - \rho^2)a + (\beta + \rho - \tau^2)\beta c]}. \tag{18}$$

Given that $\tau = \alpha'(u)$ is the marginal effect of investment on consumer utility, and $\partial r/\partial\tau > 0$, we have that when the marginal effect of investment on consumer utility increases, investment in quality becomes more intensive, as firms may recoup part of the additional consumer value through higher prices.

4.2 Capacity-Sharing Model

Under Assumption A1, the direct demands are:

$$D_i(p_i, p_j, u_i, u_j) = \frac{(a - p_i)\beta_j - (a - p_j)\rho}{\beta_i\beta_j - \rho^2}, \tag{19}$$

where $\beta_1, \beta_2 > \rho$ and $\beta_1\beta_2 > \rho^2$. From the first-order condition regarding p_i, we can determine $p_i^r = f_{p_i}(p_j, u_j; u_i)$, for $j \neq i$; specifically, we find:

$$p_i^r = \frac{a + c + \tau\big[(a + c)\beta - \rho\big(a - p_j\big)\big]u_j}{2\big(1 + \beta\tau u_j\big)}. \tag{20}$$

Hence, $\partial p_i^r/\partial u_i = 0$ and $\partial p_i^r/\partial u_j = -\tau\rho\big(a - p_j\big)/\Big[2\big(1 + \beta\tau u_j\big)^2\Big] < 0$. Drawing from the analysis presented earlier, this model shows that firms lack the incentive to increase prices upon increasing their own investment, as the demand effect neutralizes the revenue effect. Firms are incentivized to reduce their prices, however, when their competitors increase their investments.

The equilibrium is characterized by the set of first-order conditions at the symmetric solution (p, u), formulated by the equations:

$$D_i(p, p, u, u) + (p - c)\frac{\partial D_i(p, p, u, u)}{\partial p_i} = 0 \tag{21}$$

$$(p - c)\frac{\partial D_i(p, p, u, u)}{\partial u_i} - u = 0 \tag{22}$$

, where

$$\frac{\partial D_i(p, p, u, u)}{\partial p_i} = -\frac{\beta\tau u + 1}{\tau u\left[\left(\beta + \frac{1}{\tau u}\right)^2 - \rho^2\right]} < 0 \tag{23}$$

and

$$\frac{\partial D_i(p, p, u, u)}{\partial u_i} = \frac{(1 + \beta\tau u)(a - p)\tau}{[1 + u(\beta - \rho)\tau][1 + u(\beta + \rho)\tau]^2} > 0. \tag{24}$$

Given the non-linear nature of the system in terms of u, the equilibrium cannot be obtained explicitly for the price and investment. However, by solving the FOC related to price, equation (21), and using (*23*), we determine the price as a function of investment level at the symmetric equilibrium:

$$p = \frac{\tau[(a + c)\beta - a\rho]u + a + c}{2 + \tau(2\beta - \rho)u}. \tag{25}$$

This confirms our best-response analysis indicating that $dp/du < 0$. At the equilibrium, $\eta_\Gamma = 2$, but we cannot obtain explicit expressions for η_{D_p} and η_{D_u}, except when $c = 0$, where $\eta_{D_p} = 1$, thus making investment intensity contingent on the demand's investment elasticity: $\mathrm{r} = \eta_{D_u}/2$. More generally, we can establish that

$$r = \frac{a - p}{2p[1 + \tau(\beta + \rho)u]}, \tag{26}$$

where p and u are given by (22) and (25).

4.3 Simulation: Investment intensities, Pricing, Efficiencies and Congestion

To further elaborate on our understanding of pricing and investment intensity across the quality-enhancing and capacity-sharing models, this subsection first discusses investment intensities. We then analyse how gains in investment cost efficiency affect strategies in both models. Subsequently, we explore the strategic responses of firms when confronted with a shock that increases utility responsiveness to investment, such as increased network congestion due to higher user demand, a scenario currently prevalent in the telecommunications industry.

4.3.1 Investment intensities

Here, we conduct a comparative analysis of the investment intensity between the two models. For the quality-enhancing investments model, investment intensity is given by equation (*18*). For the capacity model, we cannot derive an explicit formula. To facilitate the comparison, we graphically represent investment intensity regions on the (ρ, τ) plane, considering $a = 10, \beta = 1$ and $c = 0$. Under these values, investment intensity simplifies in the quality model to $r = \tau^2/(2 - 2\rho^2)$, and is thus increasing in both τ and ρ.

The range of ρ that we consider extends from 0.05 (representing highly independent or differentiated products) to 0.8 (representing close substitutable products). In the quality-enhancing investments model, τ influences investment in a manner analogous to how ρ affects prices. Specifically, at high ρ values, a reduction in price can lead to a firm capturing the entire market demand, potentially breaking the existence of the interior equilibrium. A similar phenomenon occurs when τ is sufficiently high, and a firm marginally increases its investment level. For $\rho = 0.8$, τ must be less than $\bar{\tau} = 0.848528$ so that the interior equilibrium exists. In the capacity model, τ values approaching zero can cause β to tend to infinity. Then, to assure interior solutions, we consider a minimum τ value of 0.01; notice however that τ can potentially take any large number.[19]

[19] Unlike the quality model, where decreasing τ towards zero diminishes the investment's impact on demand, the capacity model allows for τ to increase towards infinity, effectively nullifying the investment's effect on demand.

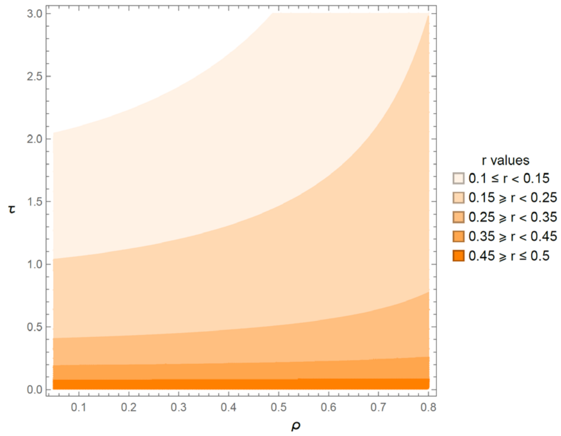

Fig. 1. Investment intensity in the capacity-sharing model.

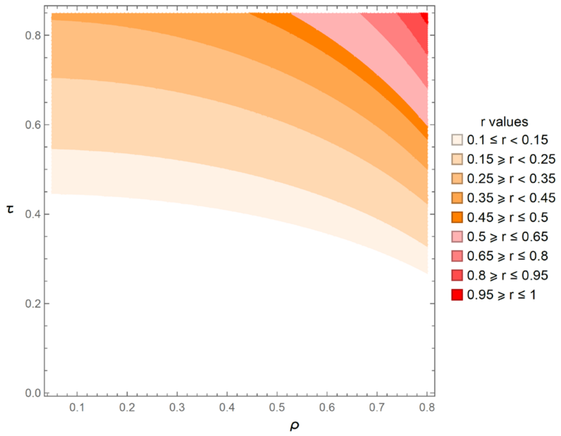

Fig. 2. Investment intensity in the quality-enhancing demand model.

In Figures 1 and 2, we depict the investment intensity regions for the capacity and quality models, respectively, revealing significant differences between them. Specifically, when consumers' utility is highly responsive to investment levels – characterized by low τ in the capacity model and high τ in the quality model – the capacity model shows a maximum investment intensity of 50%.[20] For a sufficiently low given τ value, this maximum intensity remains consistent across all ρ values. Conversely, in the quality model, investment intensity varies with ρ for a sufficiently high given τ value. Consider for example $\tau = 0.8$, at low ρ values, intensity ranges from $25 - 35\%$; for moderate values, it increases to $35 - 45\%$; and for values just above, it reaches 50%. However, investment intensity in the quality model can continue to rise as products become more closely substitutable, potentially reaching up to 100% of generated revenues. This scenario occurs when products are highly substitutable ($\rho = 0.8$), and τ reaches its maximum value ($\bar{\tau}$).

The rationale behind the results in the quality-enhancing investments model lies in the firm's ability to meet all of its demand without facing capacity constraints. When products are closely substitutable and τ is high, firms are motivated to engage in intense investment competition since a marginal improvement in the quality of their product relative to their competitors enables them to increase their market share. Unlike in the quality model, where investment directly translates into competitive advantage without the drawback of additional congestion costs, the capacity model presents a scenario where each gain in market share comes with the challenge of managing increased congestion, necessitating

[20] Recall that in the capacity model τ_i, or equivalently, t_i, is an inverse measure of efficiency of investment relative to congestion costs.

further investment to expand capacity if the firm wishes to maintain its newly acquired market share.

Investments and products substitutability. In both models, revenues (and profits) decrease as ρ increases, as a higher ρ value intensifies price competition. This effect, while leading to an increase of r in both models, results in a more pronounced decrease in prices with rising ρ in the quality model compared to the capacity model, which faces limitations in accommodating additional demand. Similarly, beginning with highly differentiated products, investment decreases with ρ in both models, due to the diminishing return on investment. However, a distinctive feature of the quality model, as opposed to the capacity model, is that investment hits its minimum value at $\rho = 0.5$. Beyond this point, investment in the quality model (and consequently, demand) begins to increase with ρ, driven by enhanced incentives to compete based on quality rather than price, which are already competitive. While investment in the quality-enhancing investments model is U-shaped with respect to ρ, it decreases monotonically in the capacity model. This shift towards tougher competition in quality investments is facilitated by the absence of capacity constraints, allowing firms to fully meet market demand.

4.3.2 Investments and Pricing

Now, we illustrate how increases in industry-wide investments (via efficiency gains in investment) have differing effects on prices and investment levels under the two models . We aim to explore the behaviour of these models under varying conditions of product substitutability and therefore competition intensity. For this purpose, we consider a scenario where $\beta = 1$ and the product differentiation parameter ρ varies from 0.2 (indicating highly independent goods) to 0.6 (indicating highly substitutable products). Establishing a common baseline for comparing the two models is challenging, given that τ influences them differently, making it impractical for comparison purposes to set the same τ value. To address this issue, we assume a τ value that yields a similar investment intensity level when competition intensity is low (i.e., products are highly differentiated). For $\rho = 0.2$ and two firms, for example, the investment intensity in both models is 10%, with $\tau = 1.5$ in the capacity model and $\tau = 0.5$ in the quality-enhancing investments model. We assume $a = 10$ and $c = 1$.

We compare the benchmark case, where $\Gamma = u^2/2$, to a case of greater investment cost efficiency, $u^2/4$ (the rest of parameters remain constant in the two cases). The focus is on examining the consequences on prices following a reduction in investment costs, which implies an increase in investment across all firms in the industry. Such a positive shift in investment costs could result from innovation or efficiencies related to a merger, such as lower capital costs or synergies from a restructuring of firms.

$\tau = 0.5$	$\rho = 0.2$	$\rho = 0.3$	$\rho = 0.4$	$\rho = 0.5$	$\rho = 0.6$
$\%\Delta p$	12.33	11.80	11.37	11.02	10.68
$\%\Delta q$	15.06	14.62	14.37	14.29	14.37
$\%\Delta u$	130.12	129.24	128.74	128.57	128.74

Table 1. Quality-enhancing investments model.

$\tau = 1.5$	$\rho = 0.2$	$\rho = 0.3$	$\rho = 0.4$	$\rho = 0.5$	$\rho = 0.6$
$\%\Delta p$	-0.51	-0.87	-1.33	-1.92	-2.71
$\%\Delta q$	6.35	6.39	6.49	6.65	6.87
$\%\Delta u$	31.27	31.31	31.39	31.52	31.70

Table 2. Capacity-sharing model.

Tables 1 and 2 display the percentage change in key variables of interest (price, quantity and investment) at equilibrium. Table 1, which refers to the quality-enhancing investments model, shows that a 50% improvement in investment cost efficiency significantly stimulates investment, which increases by more than 100%. This, in turn, leads to higher equilibrium prices throughout the industry, with price increasing by more than 10%, as well as increased demand since the investment also shifts demand outward and expands the market. On the other hand, Table 2 relates to the capacity-sharing model. In this scenario, we observe that a 50% efficiency improvement in investment costs results in an investment increase that is about 30%. Unlike in the quality-enhancing investments model, equilibrium prices decrease as investment increase, in line with our prior analysis.

5 Merger analysis

In this section, we first present a case of competition among n independent firms, which serves as our benchmark. We then present a scenario where two of these n firms merge. We use p_{-i} and u_{-i} to represent the vector of prices and investments, respectively, excluding those set by firm i. Then, $D_i(p_i, p_{-i}, u_i, u_{-i})$ denotes the demand faced by firm i when it sets its own price and investment level at p_i and u_i. For a given level of investments, a firm's demand decreases with its own price and increases with its rivals' prices. And, for a given level of prices, an increase in investment increases the quality of its product (in a quality model) or reduces congestion costs (in a capacity model), and as a result, increases its own demand.

Benchmark scenario. In the no merger scenario, each firm i addresses the following maximisation problem:

$$\max_{p_i, u_i} \pi_i = (p_i - c_i) D_i(p_i, p_{-i}, u_i, u_{-i}) - \Gamma_i(u_i). \qquad (27)$$

The first-order condition for the pricing and investment decisions are (for $i = 1, \ldots, n$):

$$\frac{\partial \pi_i}{\partial p_i} = D_i(p_i, p_{-i}, u_i, u_{-i}) + (p_i - c_i) \frac{\partial D_i(p_i, p_{-i}, u_i, u_{-i})}{\partial p_i} = 0 \qquad (28)$$

$$\frac{\partial \pi_i}{\partial u_i} = (p_i - c_i) \frac{\partial D_i(p_i, p_{-i}, u_i, u_{-i})}{\partial u_i} - \Gamma_i{}'(u_i) = 0. \qquad (29)$$

In the ensuing analysis, we assume that the firm's problem is well-behaved, meaning that the profit function π_i fulfils standard assumptions, ensuring the existence of a unique regular, symmetric interior equilibrium (Vives, 1999). Thus, the solution to the system of first-order conditions gives the equilibrium values of the price and investment for each firm in the benchmark.

Merger scenario. Consider a situation where a merger occurs between firm j and firm k, in the absence of efficiencies, the new entity faces the following problem:

$$\max_{p_j, u_j, p_k, u_k} (p_j - c_j) D_j(p_j, p_{-j}, u_j, u_{-j}) + (p_k - c_k) D_k(p_k, p_{-k}, u_k, u_{-k}) - \Gamma_j(u_j) - \Gamma_k(u_k),$$

while the rest of firms $i = 1, \ldots, n$ with $i \neq j, k$, maximise π_i. In the Appendix, we provide the expressions for the first-order conditions of the merger, along with the formulas for welfare analysis (consumer surplus and total surplus).

5.1 Efficiencies

Mergers are strategic moves that can lead to significant efficiencies for the involved firms. These efficiencies might include:

(i) *Operational efficiency,* which refers to more efficient production process allowing the merger to benefit from economies of scale, better utilisation of resources, and streamlined operations, leading to a reduction in cost per unit. This can be easily represented by pre-multiplying the marginal cost of the merging firms by a parameter $x \leq 1$;

(ii) *Investment cost efficiency,* cost savings can be a significant outcome of mergers due to reductions in overhead expenses, elimination of duplicated costs in capacity expansion efforts, greater bargaining power with suppliers, and lower cost of capital due to the improved financial health of the combine entity. In our model, this is represented by pre-multiplying the investment cost functions by $(1-d)$ with $0 \leq d < 1$, making the investment cost for the merger equal to

$$(1-d)\left(\Gamma_j(u_j) + \Gamma_k(u_k)\right). \tag{30}$$

(iii) *Network allocation efficiency,* particularly relevant in network industries like telecommunications, leads to better allocation of network resources, assets, or sites, and spectrum efficiencies. The merged entity benefits from network optimization and shared resources, which improves network capacity. This can be modelled by an increase in the capacity obtained per unit of investment: to achieve a given level of capacity, the merging firms need to invest less. This is expressed as follows:

$$\Phi(u_j) = 1/(\sigma t_j u_j) \text{ and } \Phi(u_k) = 1/(\sigma t_k u_k) \text{ with } \sigma \geq 1. \tag{31}$$

With these efficiencies, the merger maximizes the following expression:

$$\begin{aligned}\phi_{jk} = (p_j - x_j c_j) D_j(p_j, p_{-j}, u_j, u_{-j}) + (p_k - x_k c_k) D_k(p_k, p_{-k}, u_k, u_{-k}) \\ - (1-d)\left(\Gamma_j(u_j) + \Gamma_k(u_k)\right),\end{aligned} \tag{32}$$

with $x_j, x_k \leq 1$, $0 \leq d < 1$, $\beta_j = \hat{\beta}_j + 1/(\sigma \tau_j u_j)$, $\beta_k = \hat{\beta}_k + 1/(\sigma \tau_k u_k)$, and $\sigma \geq 1$.

5.2 Numerical analysis: quality-enhancing investments model vs. capacity-sharing model

In this section, we present numerical results for a merger analysis of the two models discussed throughout the paper, with the primary goal of comparing the effects of a merger on prices and consumer surplus when the merging firms experience efficiencies that lead them to increase investments. Before exploring this issue, we first comment on the effects of a merger in the two models in the absence of efficiencies. We look at post-merger changes investment intensity and changes in price, investment, profits, consumer welfare and total welfare shown in the tables below. The letter 'I' refers to the firms inside the merger, and 'O' to the outside firms.

As we did in the previous section, we assume $\tau = 0.5$ in the quality model and $\tau = 1.5$ in the capacity-sharing model. As a result, in both models with three firms, the investment intensity increases from 10% to 13% when $\rho = 0.2$.[21]

[21] Prices, investment levels, quantities, profits, consumer surplus and welfare are provided in Tables 9 and 10 in the Appendix.

$\tau = 0.5$					
ρ	0.2	0.3	0.4	0.5	0.6
r	**0.13**	0.15	0.16	0.19	0.23
$\%\Delta pI$	8.92	13.68	18.61	23.64	28.50
$\%\Delta pO$	2.19	4.68	8.03	12.40	18.17
$\%\Delta uI$	-9.23	-12.56	-15.28	-17.57	-19.69
$\%\Delta uO$	2.19	4.68	8.03	12.40	18.17
$\%\Delta\pi I$	1.42	3.29	6.04	9.76	14.58
$\%\Delta\pi O$	4.43	9.57	16.71	26.34	39.65
$\%\Delta CS$	-10.38	-12.87	-14.05	-14.09	-13.06
$\%\Delta W$	-3.52	-4.12	-4.16	-3.77	-3.02

$\tau = 0.8$					
ρ	0.2	0.3	0.4	0.5	0.6
r	**0.34**	0.37	0.41	0.48	0.58
$\%\Delta pI$	6.16	9.42	12.26	13.58	9.66
$\%\Delta pO$	3.57	7.78	14.01	23.65	41.49
$\%\Delta uI$	-11.54	-15.83	-19.82	-24.28	-31.46
$\%\Delta uO$	3.57	7.78	14.01	23.65	41.49
$\%\Delta\pi I$	2.08	4.66	8.24	12.47	14.36
$\%\Delta\pi O$	7.26	16.16	29.99	52.89	100.20
$\%\Delta CS$	-12.29	-14.75	-15.50	-14.64	-11.63
$\%\Delta W$	-4.77	-5.37	-5.15	-4.19	-2.19

Table 5. Quality-enhancing investments model with 3 firms and no efficiencies.

$\tau = 1.5$					
ρ	0.2	0.3	0.4	0.5	0.6
r	**0.13**	0.14	0.16	0.17	0.19
$\%\Delta pI$	7.50	11.24	15.00	18.78	22.54
$\%\Delta pO$	1.17	2.46	4.14	6.17	8.53
$\%\Delta uI$	-5.11	-7.10	-8.81	-10.28	-11.54
$\%\Delta uO$	0.91	1.90	3.20	4.78	6.66
$\%\Delta\pi I$	0.80	1.82	3.26	5.14	7.46
$\%\Delta\pi O$	2.66	5.66	9.67	14.71	20.90
$\%\Delta CS$	-8.31	-10.84	-12.56	-13.60	-14.01
$\%\Delta W$	-2.84	-3.56	-3.93	-4.01	-3.86

$\tau = 0.22$					
ρ	0.2	0.3	0.4	0.5	0.6
r	**0.34**	0.35	0.35	0.36	0.37
$\%\Delta pI$	3.27	4.81	6.30	7.73	9.12
$\%\Delta pO$	0.36	0.75	1.25	1.85	2.53
$\%\Delta uI$	-3.56	-5.05	-6.40	-7.63	-8.78
$\%\Delta uO$	0.41	0.86	1.43	2.10	2.88
$\%\Delta\pi I$	0.27	0.57	0.97	1.44	1.99
$\%\Delta\pi O$	1.05	2.22	3.72	5.55	7.66
$\%\Delta CS$	-5.24	-7.21	-8.86	-10.24	-11.40
$\%\Delta W$	-2.14	-2.89	-3.47	-3.93	-4.28

Table 6. Capacity-sharing model with 3 firms and no efficiencies.

In both models, a merger without efficiencies detrimentally impacts consumer surplus, primarily due to the increased prices from all firms and reduced investments by the merging firms.[22] The quality model predicts higher price increases compared to the capacity-sharing model, for both the merging firms and non-merging firms. The quality model predicts even larger price increases for non-merging firms when products are highly substitutable and consumer utility is highly sensitive to the level of investment. Similarly, the reduction in investment by merging firms is more pronounced in the quality model than in the capacity model; however, the quality model also predicts that non-merging firms will increase their investments by more compared to the capacity-sharing model. Nevertheless, there are no remarkable differences in the reduction in consumer surplus, although the negative impact on consumer surplus is slightly less pronounced in the capacity model except when products are highly substitutable and investment intensity is low.

[22] This result is in line with the findings of Federico et al. (2018), and Motta and Tarantino (2021).

The quality model exhibits more drastic variations compared to the no-merger scenario when products are less differentiated. For example, with three firms and high investment intensity, prices and investments of the outsider firm increase by more than 41%, and the investment of insider firms decreases by 31%. The capacity-sharing model, however, shows more moderate results: prices and investment of outsider firms increase by 2.5% and 2.8%, respectively, while the investment of insider firms falls by 8.7%.

Finally, when we extend both models to four firms (Tables 11 and 12 in Appendix), we observe the expected (but significant) mitigating effect of increased competition on the detrimental outcomes from the merger, and that the impact of a merger on prices and investments is softened in both models. [23]

Efficiencies. In this subsection, we assume that the efficiencies realized by the merged firms reduce their investment cost by 25%, which is significant enough to increase the investment levels of the merged entities compared to the benchmark case across all scenarios (except when products are close substitutes in the capacity-sharing model with low investment intensity). Consistent with our analysis on investment and pricing, we find that the increase in both prices and investment is significantly higher in the quality model than in the capacity model (Tables 7 and 8 present percentage changes; Tables 13 and 14 in Appendix present absolute values of the variables).

$\tau = 0.5$						$\tau = 0.8$					
ρ	0.2	0.3	0.4	0.5	0.6	ρ	0.2	0.3	0.4	0.5	0.6
r	**0.13**	0.15	0.16	0.19	0.23	r	**0.34**	0.37	0.41	0.48	0.58
$\%\Delta pI$	13.61	18.47	23.71	29.32	35.22	$\%\Delta pI$	34.88	38.70	44.16	51.65	63.32
$\%\Delta pO$	1.27	3.30	6.12	9.73	14.29	$\%\Delta pO$	-3.84	-3.29	-2.10	-1.07	-2.74
$\%\Delta uI$	26.23	21.51	17.82	14.95	12.69	$\%\Delta uI$	72.93	64.14	58.42	55.53	57.04
$\%\Delta uO$	1.27	3.30	6.12	9.73	14.29	$\%\Delta uO$	-3.84	-3.29	-2.10	-1.07	-2.74
$\%\Delta \pi I$	5.70	7.50	10.33	14.36	19.88	$\%\Delta \pi I$	29.30	31.93	38.04	49.68	75.63
$\%\Delta \pi O$	2.55	6.71	12.61	20.41	30.63	$\%\Delta \pi O$	-7.52	-6.46	-4.16	-2.13	-5.40
$\%\Delta CS$	-6.11	-9.25	-10.87	-11.23	-10.45	$\%\Delta CS$	14.80	6.94	2.60	0.75	0.95
$\%\Delta W$	-0.34	-1.35	-1.69	-1.53	-1.00	$\%\Delta W$	15.84	11.86	9.74	8.90	9.20

Table 7. Quality-enhancing investments model with 3 firms and efficiencies ($d = 0.25$).

[23] Specifically, the price increase following a merger diminishes, and the negative impact on the investment of the merging firms becomes less severe. Simultaneously, the positive impact on the investment of non-merging firms also decreases. The overall outcome is a significant reduction in the adverse effects of the merger on consumer surplus: In the quality model and low investment intensity scenario, the decrease in consumer surplus shifts from -10% to -6% for $\rho = 0.2$ and from -13% to -5% for $\rho = 0.6$; similarly, in the capacity model, the fall in consumer surplus changes from -8.3% to -5% for $\rho = 0.2$ and from -14% to -6.4% for $\rho = 0.6$.

	$\tau = 1.5$						$\tau = 0.22$				
ρ	0.2	0.3	0.4	0.5	0.6	ρ	0.2	0.3	0.4	0.5	0.6
r	**0.13**	0.14	0.16	0.17	0.19	r	**0.34**	0.35	0.35	0.36	0.37
$\%\Delta pI$	7.47	11.18	14.89	18.60	22.26	$\%\Delta pI$	3.27	4.80	6.28	7.71	9.09
$\%\Delta pO$	0.75	1.80	3.20	4.89	6.82	$\%\Delta pO$	-0.71	-0.81	-0.79	-0.67	-0.47
$\%\Delta uI$	6.26	4.06	2.19	0.61	-0.73	$\%\Delta uI$	23.05	21.04	19.26	17.66	16.22
$\%\Delta uO$	0.65	1.54	2.73	4.21	5.97	$\%\Delta uO$	-0.74	-0.75	-0.60	-0.31	0.09
$\%\Delta \pi I$	4.61	5.71	7.28	9.33	11.89	$\%\Delta \pi I$	21.45	21.64	22.02	22.56	23.26
$\%\Delta \pi O$	1.80	4.34	7.81	12.19	17.51	$\%\Delta \pi O$	-1.98	-2.16	-1.95	-1.42	-0.63
$\%\Delta CS$	-6.79	-9.37	-11.12	-12.15	-12.55	$\%\Delta CS$	4.43	1.92	-0.18	-1.94	-3.41
$\%\Delta W$	-0.91	-1.73	-2.19	-2.35	-2.25	$\%\Delta W$	9.38	8.04	6.94	6.05	5.31

Table 8. Capacity-sharing model with 3 firms and efficiencies ($d = 0.25$).

In industries with low investment intensity, a 25% reduction in investment costs results in higher quality/capacity. However, even though such increased investment benefits consumers, it may not be sufficient to fully offset the price increases caused by the merger (therefore consumer welfare may not necessarily increase). In industries with high investment intensity, total surplus increases across all considered ρ parameter ranges in both models. In the quality model, consumer surplus also increases for all ρ values, though it diminishes as products become more substitutable. In the capacity model, consumer surplus also rises when products range from moderately to highly differentiated.

6 Model calibration: Example with 4 firms

In this section, we consider a market involving four firms, specifically focusing on a merger between firms 1 and 2. Additionally, we will discuss how to calibrate this model using prices, quantities, diversion ratios, and levels of investment and congestion specific to each firm.

6.1 The model

We adopt the following specification for the investment cost function:

$$\Gamma_i(u_i) = \omega_i u_i^{\kappa_i}, \text{ where } w_i > 0 \text{ and } \kappa_i \geq 1. \quad (33)$$

This power function satisfies the assumptions,[24] and it is chosen for both technical and economic reasons.[25] The coefficient ω_i converts the units of investment in capacity into a monetary cost per unit period.

[24] Where $\Gamma_i(0) = 0$, $\Gamma_i' > 0$, and $\Gamma_i'' \geq 0$.

[25] Technically, a higher value of κ_i enhances the concavity of the profit function, thereby increasing the likelihood that the second-order conditions are met, especially when t_i is small where an interior equilibrium may not exist. Economically, as previously discussed, there may be compelling reasons

In the capacity-sharing model, the representative consumer problem, with utility function given by (5), yields the following inverse demand system:

$$\begin{aligned} p_1(\boldsymbol{q}) &= a_1 - \beta_1 q_1 - \rho_{12} q_2 - \rho_{13} q_3 - \rho_{14} q_4 \\ p_2(\boldsymbol{q}) &= a_2 - \rho_{12} q_1 - \beta_2 q_2 - \rho_{23} q_3 - \rho_{24} q_4 \\ p_3(\boldsymbol{q}) &= a_3 - \rho_{13} q_1 - \rho_{23} q_2 - \beta_3 q_3 - \rho_{34} q_4 \\ p_4(\boldsymbol{q}) &= a_4 - \rho_{14} q_1 - \rho_{24} q_2 - \rho_{34} q_3 - \beta_4 q_4, \end{aligned} \tag{34}$$

where $\beta_i = \hat{\beta}_i + 1/(\tau_i u_i)$ for all $i = 1, \ldots, 4$. Inverting the inverse demand system yields the following direct demands:

$$D_1(p_1, p_{-1}, u_1, u_{-1}) = A_1 + B_1 p_1 + M_{12} p_2 + M_{13} p_3 + M_{14} p_4$$

$$D_2(p_2, p_{-2}, u_2, u_{-2}) = A_2 + M_{12} p_1 + B_2 p_2 + M_{23} p_3 + M_{24} p_4$$

$$D_3(p_3, p_{-3}, u_3, u_{-3}) = A_3 + M_{13} p_1 + M_{23} p_2 + B_3 p_3 + M_{34} p_4$$

$$D_4(p_4, p_{-4}, u_4, u_{-4}) = A_4 + M_{14} p_1 + M_{24} p_2 + M_{34} p_3 + B_4 p_4,$$

where A_i, B_i and M_{ij} are complex functions of a_i, β_i (and, consequently, u_i), and ρ_{ij}. The direct demands are linear in prices but non-linear in investment levels. They exhibit downward sloping and symmetric cross effects, as the Hessian of $\widetilde{U}$ is negative definite and symmetric.

In the benchmark scenario (pre-merger), each firm $i = 1, \ldots, 4$ maximises the following expression:

$$\pi_i = (p_i - c_i) D_i(p_i, p_{-i}, u_i, u_{-i}) - \omega_i u_i^{\kappa_i}, \tag{35}$$

where $D_i(p_i, p_{-i}, u_i, u_{-i})$ is as given by the above direct demand system. The pre-merger equilibrium is the solution to the 8-equation system of first-order conditions derived from equations (28) and (29).

Post-merger, the merged entity maximises ϕ_{jk} as given by equation (32), while the remaining firms each maximise π_i.[26]

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to adopt a strictly increasing rate of growth for the investment cost function, which is attainable in this framework with $\kappa_i > 1$. Additionally, this function's flexibility allows u_i to serve as a proxy variable rather than just representing investment levels, accommodating various interpretations that may be needed for quantitative merger analyses.

[26] The equilibrium is the solution to system of first-order conditions outlined in equations (41) to (44) (provided in Appendix), where $j = 1$ and $k = 2$, and equations (28) and (29) for $i = 3,4$.

6.2 Calibration

In this subsection, we explain the calibration of the model to quantify merger effects in industries where network capacity, such as in fixed or mobile communications, plays a critical role. We aim to find values for unknown parameters so that the equilibrium of the non-cooperative game reflects observed market values.

Suppose we have observable data including average monthly charges, quantities demanded, and margins: $\bar{p}_1, \bar{p}_2, \bar{p}_3, \bar{p}_4, \bar{q}_1, \bar{q}_2, \bar{q}_3, \bar{q}_4, \bar{m}_1, \bar{m}_2, \bar{m}_3$ and $\bar{m}_4$. First, we will calibrate for marginal costs and the parameters B_i of direct demands. And, after that, using observed diversion ratios, $\bar{D}_{ji}$, we will calibrate the parameters M_{ji} of direct demands. Lastly, the investment cost function will be calibrated with observed investment levels $\bar{u}_1, \bar{u}_2, \bar{u}_3$ and $\bar{u}_4$.

Marginal costs. The marginal costs are calibrated using the margin definition and observed charges and margins: $\bar{m}_i = (\bar{p}_i - c_i)/\bar{p}_i$. The calibrated marginal costs are then:

$$c_i^c = \bar{p}_i(1 - \bar{m}_i),$$

where, from now on, the superscript c denotes a calibrated value.

Parameters $\boldsymbol{B_i}$. For parameters B_i, using the first-order condition with respect to price, equation (28), we can write: $B_i = \frac{\partial D_i}{\partial p_i} = -D_i(p_i, p_{-i}, u_i, u_{-i})/(\bar{p}_i - c_i^c)$. Substituting observed values into this equation allows us to calibrate B_i for all $i = 1, \ldots, 4$:

$$B_i^c = -\frac{\bar{q}_i}{\bar{p}_i \bar{m}_i}.$$

Parameters $\boldsymbol{M_{ji}}$. Let

$$D_{ij} = -\left(\frac{\partial D_j / \partial p_i}{\partial D_i / \partial p_i} \right)$$

be the diversion ratio from firm i to firm j. Solving this for $\partial D_j / \partial p_i$, we have

$$M_{ji} = \frac{\partial D_j}{\partial p_i} = -D_{ij}\left(\frac{\partial D_i}{\partial p_i}\right) = -D_{ij}B_i. \qquad (36)$$

If we have limited observed diversion ratios, such as: $\bar{D}_{12}, \bar{D}_{13}, \bar{D}_{14}, \bar{D}_{23}, \bar{D}_{24}$ and $\bar{D}_{34}$, we can use Equation (36) to calibrate the corresponding M_{ji}. For instance, $M_{12}^c = -\bar{D}_{21}B_2^c$, $M_{13}^c = -\bar{D}_{31}/B_3^c$, $M_{14}^c = -\bar{D}_{41}B_4^c$, $M_{23}^c = -\bar{D}_{32}B_3^c$, $M_{24}^c = -\bar{D}_{42}B_4^c$ and $M_{34}^c = -\bar{D}_{43}B_4^c$. When surveys provide diversion ratios in both directions ($\bar{D}_{ij}$ and $\bar{D}_{ji}$ with $j \neq i$), we can calibrate two values for each M_{ji}. Considering M_{12} and the symmetry of the demand system, we get: M_{12}: $M_{12}^c = \partial D_1/\partial p_2 = -\overline{D}_{21}B_2^c$ and $M_{12}^c = \partial D_2/\partial p_1 = -\overline{D}_{12}B_1^c$.

Suppose we have the full set of observed diversion ratios; we get two potential calibrated values for each M_{ji}: $M_{ji}^c \in \{-\overline{D}_{ij}B_i^c, -\overline{D}_{ji}B_j^c\}$. Frequently, these values may not be equal, leading to an over-identified model: with four firms, there exist six M_{ji} parameters, each with two potential calibrated values, resulting in $2^6 = 64$ possible combinations of M_{ij} values for calibrating the model. Let combination k (where $k = 1, \dots, 64$) be represented by $\mathbb{C}^k = \left(M_{12}^{c_k}, M_{13}^{c_k}, M_{14}^{c_k}, M_{23}^{c_k}, M_{24}^{c_k}, M_{34}^{c_k}\right)$, with $M_{ji}^{c_k} \in \{-\overline{D}_{ij}B_i^c, -\overline{D}_{ji}B_j^c\}$ and $k = 1, \dots, 64$. We have:

$$\begin{aligned} \mathbb{C}^1 &= (-\overline{D}_{21}B_2^c, -\overline{D}_{31}B_3^c, -\overline{D}_{41}B_4^c, -\overline{D}_{32}B_3^c, -\overline{D}_{42}B_4^c, -\overline{D}_{43}B_4^c) \\ \mathbb{C}^2 &= (-\overline{D}_{21}B_2^c, -\overline{D}_{31}B_3^c, -\overline{D}_{41}B_4^c, -\overline{D}_{32}B_3^c, -\overline{D}_{42}B_4^c, -\overline{D}_{34}B_3^c) \\ &\vdots \\ \mathbb{C}^{64} &= (-\overline{D}_{12}B_1^c, -\overline{D}_{13}B_1^c, -\overline{D}_{14}B_1^c, -\overline{D}_{23}B_2^c, -\overline{D}_{24}B_2^c, -\overline{D}_{34}B_3^c). \end{aligned} \tag{37}$$

The first combination employs $M_{ji}^c = -\overline{D}_{ij}B_i^c$ for all $j \neq i$, while the last uses $M_{ji}^c = -\overline{D}_{ji}B_j$. Over-identification in this context arises for two primary reasons: firstly, and obviously, because the available information exceeds what is necessary to calibrate the model; and secondly, because the model, being a simplification of reality, cannot fully capture all the underlying complexities that might reconcile conflicting pieces of information. It is important to note that such over-identification issues are typical in IO models where demand functions, derived from the consumer's problem, are symmetric.

To address this over-identification, we may select the combination $\mathbb{C}^k$ that yields the lowest sum of squared errors between the model-implied and observed diversion ratios. From Equation (36), we have that $D_{ij} = -M_{ji}/B_i$. Therefore, for a given combination k, the sum of squared errors for parameter M_{ji} can be calculated as:

$$\varepsilon_{ij}^k = \left(-\frac{M_{ji}^{c_k}}{B_i^c} - \overline{D}_{ij}\right)^2 + \left(-\frac{M_{ij}^{c_k}}{B_j^c} - \overline{D}_{ji}\right)^2.$$

In each combination k, one of the two components of ε_{ij} will be zero, while the other will generate an error. We can then compute a measure of error for each combination k:

$$err_k = \varepsilon_{12}^k + \varepsilon_{13}^k + \varepsilon_{14}^k + \varepsilon_{23}^k + \varepsilon_{24}^k + \varepsilon_{34}^k,$$

and calibrate the M_{ji} parameters based on the combination $\mathbb{C}^k$ that produces the lowest error measure (i.e., that solves $\min\{err_1, \dots, err_{64}\}$) by using the diversion ratios of the corresponding set $\mathbb{C}^k$ in equation (37).

Utility function calibration. For welfare analysis we need to calibrate the parameters of the utility function: a_i, β_i, ρ_{ij}. Initially, we focus on calibrating the values of A_i. By inserting the calibrated parameters B_i^c and M_{ji}^c into the direct demand system, along with observed prices and quantities demanded, and then solving for A_i we can obtain calibrated values as follows:

$$A_1^c = \bar{q}_1 - B_1^c\bar{p}_1 - M_{12}^c\bar{p}_2 - M_{13}^c\bar{p}_3 - M_{14}^c\bar{p}_4$$

$$A_2^c = \bar{q}_2 - M_{12}^c\bar{p}_1 - B_2^c\bar{p}_2 - M_{23}^c\bar{p}_3 - M_{24}^c\bar{p}_4$$

$$A_3^c = \bar{q}_3 - M_{13}^c\bar{p}_1 - M_{23}^c\bar{p}_2 - B_3^c\bar{p}_3 - M_{34}^c\bar{p}_4$$

$$A_4^c = \bar{q}_4 - M_{14}^c\bar{p}_1 - M_{24}^c\bar{p}_2 - M_{34}^c\bar{p}_3 - B_4^c\bar{p}_4.$$

It is important to note that A_i, B_i and M_{ji} are complex functions of a_1, a_2, a_3, a_4, β_1, β_2, β_3, β_4, ρ_{12}, ρ_{13}, ρ_{14}, ρ_{23}, ρ_{24} and ρ_{34}. By setting $A_1 = A_1^c$, $A_2 = A_2^c$, $A_3 = A_3^c$, $A_4 = A_4^c$, $B_1 = B_1^c$, $B_2 = B_2^c$, $B_3 = B_3^c$, $B_4 = B_4^c$, $M_{12} = M_{12}^c$, $M_{13} = M_{13}^c$, $M_{14} = M_{14}^c$, $M_{23} = M_{23}^c$, $M_{24} = M_{24}^c$, $M_{34} = M_{34}^c$, we establish a system of 14 equations with 14 unknown variables, solvable through numerical methods. Solving this system provides the calibrated values a_i^c, β_i^c and ρ_{ij}^c.

Congestion costs. Congestion costs are modelled through the function $l_i(q_i, u_i) = \mu q_i/(\theta_i u_i) = q_i/(t_i u_i)$, which is captured by the β_i parameter in the utility function: $\beta_i = \hat{\beta}_i + 1/(\tau_i u_i)$ with $\tau_i \equiv t_i/2$. For each firm i, the calibrated value β_i^c and the observed investment level $\bar{u}_i$ are known, but we face a challenge with one equation and two unknown variables: $\hat{\beta}_i$ and τ_i (or equivalently, t_i). We propose three strategies to tackle this under-identification:

1. *Calibrating from observed congestion costs:* If data on consumers' average congestion costs for each firm i ($\bar{l}_i$) are available or, alternatively, can be derived from surveys on consumers' willingness to pay for related attributes, we can calibrate t_i by solving $\bar{l}_i = \bar{q}_i/(t_i\bar{u}_i)$ for t_i, yielding t_i^c and consequently, $\tau_i^c = t_i^c/2$. This leads to the calibrated value of $\hat{\beta}_i$:

$$\hat{\beta}_i^c = \beta_i^c - \frac{1}{\tau_i^c\bar{u}_i}. \tag{38}$$

2. *Using elasticities for calibration:* Focusing on the elasticity of the absolute value of the inverse demand function's slope, β_i, we can calibrate τ_i. The elasticity E_{β_i} is given by:

$$E_{\beta_i} = -\frac{\partial \beta_i}{\partial u_i}\frac{u_i}{\beta_i} = \frac{1}{\tau_i u_i \beta_i}. \quad (39)$$

Suppose that we know that a one percent increase in firm's i investment decreases the slope of its inverse demand function by $\bar{E}_{\beta_i} > 0$ percent, then we can calibrate τ_i by substituting into the above equation and solving for τ_i: $\tau_i^c = 1/\left(\bar{E}_{\beta_i}\bar{u}_i\beta_i^c\right)$. Given that inverse demand corresponds to price and is the same as the average revenue, this elasticity can be inferred from data on how variations in investment levels impact average revenue.[27] This elasticity influences the incentives to invest and determines the relative changes in prices and investment due to the merger. Given that this approach entails calibrating the parameter τ_i so that elasticity at equilibrium equals the predetermined value of $\bar{E}_{\beta_i}$, the specific level of u_i used for calibration becomes irrelevant, only the product $\tau_i u_i$ matters. Numerical simulations confirm that, although the absolute values of equilibrium prices and investment levels post-merger shift depending on the observed u_i profile employed for calibration, the impact of the merger on the relative variation of equilibrium prices and investments remains independent of this profile, relying on the predetermined levels of elasticity.

3. *Calibrating from observed congestion levels:* In the absence of direct data on congestion costs and elasticities, we may use congestion levels for each firm i (such as processing delays) as $\vartheta_i = q_i/(\theta_i u_i)$, where $\theta_i > 0$. Using this definition and observed congestion levels $\bar{\vartheta}_i$ along with known quantities and investment levels, we can calibrate the parameter θ_i using the equation $\theta_i^c = \frac{\bar{q}_i}{(\bar{\vartheta}_i\bar{u}_i)}$. To complete the calibration of congestion costs: $l_i(q_i, u_i) = \mu\left(\frac{q_i}{\theta_i u_i}\right)$, we can determine a valid value for μ (denoted $\bar{\mu}$) from consumer surveys or using proxies and conduct sensitivity analyses. This approach yields $t_i^c = \theta_i^c/\bar{\mu}$ and $\tau_i^c = \theta_i^c/(2\bar{\mu})$, enabling the calibration of $\hat{\beta}_i$. We may then calibrate the elasticities E_{β_i}, which, as previously mentioned, are critical for understanding the relative impacts of the merger on strategic variables. Notice that, by inserting $\tau_i^c = \theta_i^c/(2\bar{\mu})$ into (39) we can write

[27] The value of this calibration strategy lies not in greater identification power, but in using elasticity as an indicative economic variable to which an easily interpretable, economically meaningful value can be assigned, in contrast to other variables. Of course, when determining the elasticity value using variations in investment levels relative to average revenue, an econometric analysis should be conducted, complementing the model calibration exercise.

$E^c_{\beta_i} = (2\bar{\mu}\bar{\vartheta}_i)/(\beta^c_i \bar{q}_i)$, which does not depend on the observed u_i level used for model calibration.[28]

Investment cost function. By calibrating ω_i for each firm i, we can ensure that in the equilibrium of the non-cooperative game, the u_i levels mirror those observed in the market, and since β_i depends on u_i, we also ensure that equilibrium prices match those observed in the market.[29] This calibration involves finding the set of coefficients $(\omega_1, \omega_2, \omega_3, \omega_4)$ that satisfies the system of first-order conditions with respect to u_i, evaluated at market levels, for given values of κ_i.

Finally, revisiting our earlier discussion on the value of parameter κ_i, higher values of κ_i enhance the likelihood of the profit function satisfying the second-order conditions. And, economically, there is also substantial justification for a progressively increasing growth rate of the investment cost function. This increase is feasible within our framework when κ_i exceeds 1. Trying to calibrate the exact value of κ_i poses certain challenges. Ideally, one might consider utilising firm-level profit data for this calibration. Yet, this approach is not straightforward. Typically, available profit figures encompass a diverse array of markets, activities, and even asset sales, rendering them less reliable for precise calibration. An alternative strategy involves assessing the economic rationale behind setting κ_i equal to 1 provided second-order conditions are satisfied at that level. If second-order conditions fail at or there are economic reasons against $\kappa_i = 1$, then the model favors a value of κ_i greater than 1. In such a case, the aim is to identify a κ_i value ensuring that the firm's optimisation problem fulfils the second-order conditions across all considered or feasible parameter ranges. This calibration process might also involve a sensitivity analysis for a limited number of κ_i values exceeding 1, which can be economically justified.

7 Conclusions

Investments in network capacity affect service quality and alter competition at the retail level. Traditional models, which predominantly concentrate on cost-reducing or quality-enhancing investments, fall short in addressing industries where traffic flow and network capacity are of key relevance for consumers and competition. By incorporating a capacity-

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[28] Hence, like the previous strategy that assumes predetermined values for the elasticities, the levels of u_i used for model calibration do not influence the relative variations in p_i and u_i resulting from the merger; what matters is the calibrated E_{β_i}. This finding is further validated by numerical simulations.
[29] This is especially useful as the coefficient ω_i enables the conversion of annual investment quantities to monthly figures, or translates a multi-year investment into an optimal monthly figure.

sharing model into a representative consumer framework, we have formulated inverse demand functions that are linear in quantities. An increase in capacity investment results in a rotation of the inverse demand function, contrasting with the typical demand shift observed in standard demand-enhancing models. We argue that, as a result of this inverse demand rotation, the capacity-sharing model offers an accurate depiction of industries facing capacity constraints and congestion.

Our analysis finds significant differences in how firms adjust their pricing strategy in response to changes in their investment. Unlike the quality-enhancing investments model, where firms typically raise prices following investment increases, the capacity-sharing model reveals an incentive for firms to lower prices to leverage profitability from new demand, which incurs lesser congestion costs. Additionally, the capacity model demonstrates a consistent maximum investment intensity across varying degrees of product substitutability, provided utility is highly sensitive to investments. In contrast, the quality model shows variable investment intensity based on the degree of product substitutability, with potential investment reaching up to 100% of revenues for highly substitutable products, where intense competition in investment becomes attractive due to the lack of capacity constraints. The quality model exhibits a U-shaped investment response to the degree of product substitutability, contrasting with the capacity model's monotonic decrease.

Efficiency gains in investment lead to significant increases in investment and higher prices in the quality model. In contrast, the capacity model shows moderate increases in investment and *decreasing* prices.

Our analysis underscores significant divergences in the impact of mergers across the quality and capacity-sharing models. For instance, in a setting with three firms and high investment intensity, the quality model shows large increases in prices and investments for outsider firms by over 41%, whereas insider firms see a 31% reduction in investment. In contrast, the capacity-sharing model presents more tempered responses, with modest increases for outsider firms (2.5% in prices and 2.8% in investment) and decreases in insider firms' investment (8.7%). Despite these differences, the impact on consumer surplus from the merger is similar across both models, albeit slightly lower in the capacity model, particularly when products are moderately to highly differentiated.

When considering efficiencies from synergies which reduce investment costs, we observe more pronounced effects in the quality model, where both prices and investments significantly increase. In low investment intensity scenarios consumer welfare may still go down if the detrimental impact of price increases is larger than the gains from increasing quality or capacity. However, in high investment intensity scenarios, total surplus increases in both models, and consumer surplus increases in the quality model. The capacity model reveals an improvement in consumer surplus for moderately to highly differentiated products. Moreover, efficiencies resulting from the merger tend to be most beneficial to consumers where investments are intensive.

Our approach also provides a step-by-step guide for calibrating the capacity-sharing model using real-world observable data. This calibration process enables the quantification of the impacts of mergers on prices, demand quantities, consumer surplus, and overall welfare. Our expectation is that the proposed model and calibration methods will prove to be valuable tools for practitioners and competition authorities, especially in assessing merger proposals in industries where considerations of congestion and capacity play a crucial role.

Appendix

Merger First-Order Conditions. The first-order conditions for the merger are: $\partial\phi_{jk}/\,\partial p_j = 0$, $\partial\phi_{jk}/\,\partial p_k = 0$, $\partial\phi_{jk}/\partial u_j = 0$, $\partial\phi_{jk}/\partial u_k = 0$, that is,

$$\frac{\partial\phi_{jk}}{\partial p_j} = D_j\big(p_j, p_{-j}, u_j, u_{-j}\big) + \big(p_j - x_j c_j\big)\frac{\partial D_j\big(p_j, p_{-j}, u_j, u_{-j}\big)}{\partial p_j} + (p_k - x_k c_k)\frac{\partial D_k(p_k, p_{-k}, u_k, u_{-k})}{\partial p_j} = 0, \tag{40}$$

$$\frac{\partial\phi_{jk}}{\partial p_k} = \big(p_j - x_j c_j\big)\frac{\partial D_j\big(p_j, p_{-j}, u_j, u_{-j}\big)}{\partial p_k} + D_k(p_k, p_{-k}, u_k, u_{-k}) + (p_k - x_k c_k)\frac{\partial D_k(p_k, p_{-k}, u_k, u_{-k})}{\partial p_k} = 0, \tag{41}$$

$$\frac{\partial\phi_{jk}}{\partial u_j} = \big(p_j - x_j c_j\big)\frac{\partial D_j\big(p_j, p_{-j}, u_j, u_{-j}\big)}{\partial u_j} + (p_k - x_k c_k)\frac{\partial D_k(p_k, p_{-k}, u_k, u_{-k})}{\partial u_j} - (1-d)\Gamma_j'\big(u_j\big) = 0, \tag{42}$$

$$\frac{\partial\phi_{jk}}{\partial u_k} = \big(p_j - x_j c_j\big)\frac{\partial D_j\big(p_j, p_{-j}, u_j, u_{-j}\big)}{\partial u_k} + (p_k - x_k c_k)\frac{\partial D_k(p_k, p_{-k}, u_k, u_{-k})}{\partial u_k} - (1-d){\Gamma_k}'(u_k) = 0. \tag{43}$$

For the rest of firms $i \neq j, k = 1, \dots, n$, the first-order conditions are given by the system of equations (28) and (29).

Welfare Analysis of the Merger. Given that utility is linear in income, the model allows for a welfare analysis. The consumer surplus (CS) is defined as:

$$CS = \widetilde{U} - \sum_{i=1}^{n} p_i q_i, \tag{44}$$

while total surplus (TS) is:

$$TS = \widetilde{U} - \sum_{i=1}^{n} c_i q_i - \sum_{i=1}^{n} \Gamma_i\,(u_i). \tag{45}$$

Consider the merger scenario between firm j and firm k. Let (p_i^{nm}, u_i^{nm}) and (p_i^{m}, u_i^{m}) represent the price and investment level in the no merger and merger equilibria, respectively, for firm i (where $i = 1, \dots, n$). Additionally, let q_i^{nm} and q_i^{m}, and $\widetilde{U}^{nm}$ and $\widetilde{U}^{m}$, denote the demand and utility levels evaluated at the corresponding equilibria. The change in consumer surplus is then calculated as:

$$\Delta CS \equiv CS^m - CS^{nm} = \widetilde{U}^m - \sum_{i=1}^{n} p_i^m q_i^m - \widetilde{U}^{nm} + \sum_{i=1}^{n} p_i^{nm} q_i^{nm}, \tag{46}$$

where $\beta_i \equiv \bar{\beta}_i + 1/(\tau_i u_i)$ for all i in the no merger scenario and for all $i \neq j, k$ in the merger scenario; in the presence of network allocation efficiencies, and in the merger scenario, $\beta_j = \bar{\beta}_j + 1/\left(\sigma \tau_j u_j^m\right)$ and $\beta_k = \bar{\beta}_k + 1/(\sigma \tau_k u_k^m)$ for $j, k \neq i$. The change in total surplus is given by:

$$\begin{aligned}\Delta TS \equiv\ & TS^m - TS^{nm} \\ &= \widetilde{U}^m - \left(x_j c_j q_j^m + x_k c_k q_k^m\right) - \sum_{i \neq j,k}^{n} c_i q_i^m \\ &- \left(\Gamma_j\left(u_j^m\right) + \Gamma_k(u_k^m)\right)(1-d) - \sum_{i \neq j,k}^{n} \Gamma_i(u_i^m) - \widetilde{U}^{nm} \\ &+ \sum_{i=1}^{n} c_i q_i^{nm} + \sum_{i=1}^{n} \Gamma_i(u_i^{nm}),\end{aligned} \tag{47}$$

with $x_j, x_k \leq 1$, $0 \leq d < 1$, and $\sigma \geq 1$.

Tables. *Notation.* 'r' represents the pre-merger industry investment intensity. The letters 'c' and 'm' denote the competitive and merger scenarios, respectively, while 'I' refers to the firms inside the merger, and 'O' refers to the outside firms. 'pc' is the pre-merger equilibrium price, 'pI' is the post-merger equilibrium price for the merging firms, and 'pO' is the post-merger equilibrium price for the non-merging firms. This notation is similarly applied to 'uc', 'uI', 'uO' for investment levels, and 'qc', 'qI' and 'qo' for quantities. 'π_c' indicates the pre-merger equilibrium profit, 'ϕ' represents the sum of equilibrium profits for the two merging firms, and 'π_O' is the post-merger equilibrium profit for the non-merging firms.

$\tau = 0.5$					
ρ	0.2	0.3	0.4	0.5	0.6
r	**0.13**	0.15	0.16	0.19	0.23
pc	4.48	3.90	3.32	2.76	2.20
pI	4.88	4.43	3.94	3.41	2.83
pO	4.58	4.08	3.59	3.10	2.60
uc	2.40	2.26	2.15	2.07	2.00
uI	2.18	1.98	1.82	1.71	1.61
uO	2.45	2.37	2.33	2.33	2.36
qc	4.80	4.52	4.31	4.14	4.00
qI	4.36	3.95	3.65	3.41	3.21
qO	4.91	4.73	4.65	4.65	4.73
πc	18.62	15.06	12.00	9.27	6.80
ϕ	37.78	31.11	25.44	20.36	15.58
πO	19.45	16.50	14.00	11.72	9.50
ΔCS	-5.02	-6.32	-7.04	-7.24	-6.89
ΔW	-3.67	-3.88	-3.58	-2.98	-2.21

$\tau = 0.8$					
ρ	0.2	0.3	0.4	0.5	0.6
r	**0.34**	0.37	0.41	0.48	0.58
pc	5.51	4.73	3.99	3.29	2.61
pI	5.85	5.18	4.48	3.74	2.86
pO	5.71	5.10	4.55	4.07	3.69
uc	4.72	4.39	4.14	3.95	3.79
uI	4.18	3.70	3.32	2.99	2.60
uO	4.89	4.73	4.72	4.88	5.36
qc	5.91	5.49	5.18	4.93	4.74
qI	5.22	4.62	4.15	3.74	3.25
qO	6.12	5.92	5.90	6.10	6.71
πc	21.39	16.32	12.10	8.44	5.17
ϕ	43.67	34.16	26.20	18.99	11.82
πO	22.94	18.96	15.73	12.90	10.34
ΔCS	-9.00	-10.67	-11.22	-10.69	-8.62
ΔW	-6.56	-6.51	-5.60	-4.12	-1.96

Table 9. Quality-enhancing investments model with 3 firms and no efficiencies.

$\tau = 1.5$					
ρ	0.2	0.3	0.4	0.5	0.6
r	**0.13**	0.14	0.16	0.17	0.19
pc	4.26	3.90	3.54	3.19	2.84
pI	4.58	4.33	4.07	3.78	3.48
pO	4.31	3.99	3.69	3.38	3.08
uc	1.93	1.86	1.81	1.76	1.72
uI	1.83	1.73	1.65	1.58	1.52
uO	1.95	1.90	1.87	1.85	1.84
qc	3.29	3.12	2.98	2.86	2.77
qI	3.04	2.79	2.44	2.22	2.02
qO	3.34	3.21	2.79	2.59	2.44
πc	12.14	10.41	8.91	7.58	6.37
ϕ	24.48	21.20	18.40	15.93	13.69
πO	12.46	11.00	9.77	8.69	7.70
ΔCS	-2.36	-3.10	-3.63	-3.98	-4.17
ΔW	-1.84	-2.13	-2.19	-2.09	-1.88

$\tau = 0.22$					
ρ	0.2	0.3	0.4	0.5	0.6
r	**0.34**	0.35	0.35	0.36	0.37
pc	4.67	4.52	4.37	4.23	4.09
pI	4.83	4.74	4.65	4.55	4.46
pO	4.69	4.55	4.43	4.30	4.19
uc	2.21	2.15	2.09	2.04	1.99
uI	2.13	2.04	1.95	1.88	1.82
uO	2.22	2.16	2.12	2.08	2.05
qc	1.54	1.47	1.42	1.37	1.32
qI	1.46	1.36	1.31	1.24	1.18
qO	1.55	1.49	1.40	1.34	1.28
πc	4.76	4.36	4.01	3.69	3.41
ϕ	9.54	8.77	8.09	7.49	6.95
πO	4.81	4.46	4.16	3.90	3.67
ΔCS	-0.65	-0.87	-1.06	-1.21	-1.34
ΔW	-0.57	-0.73	-0.83	-0.90	-0.94

Table 10. Capacity-sharing model with 3 firms and no efficiencies.

$\tau = 0.5$					
ρ	0.2	0.3	0.4	0.5	0.6
r	**0.14**	0.15	0.17	0.20	0.25
$\%\Delta pI$	7.46	10.53	13.20	15.41	16.92
$\%\Delta pO$	1.69	3.25	5.08	7.17	9.66
$\%\Delta uI$	-7.89	-10.19	-11.95	-13.44	-14.97
$\%\Delta uO$	1.69	3.25	5.08	7.17	9.66
$\%\Delta \pi I$	1.22	2.56	4.23	6.14	8.24
$\%\Delta \pi O$	3.42	6.61	10.41	14.85	20.26
$\%\Delta CS$	-5.99	-6.66	-6.56	-5.96	-5.02
$\%\Delta W$	-1.86	-1.86	-1.62	-1.26	-0.87

$\tau = 0.8$					
ρ	0.2	0.3	0.4	0.5	0.6
r	**0.35**	0.38	0.44	0.51	0.63
$\%\Delta pI$	4.96	6.70	7.43	6.10	-1.11
$\%\Delta pO$	2.72	5.35	8.81	13.76	22.94
$\%\Delta uI$	-10.04	-13.31	-16.44	-20.43	-28.08
$\%\Delta uO$	2.72	5.35	8.81	13.76	22.94
$\%\Delta \pi I$	1.68	3.36	5.21	6.57	3.94
$\%\Delta \pi O$	5.52	10.99	18.39	29.40	51.14
$\%\Delta CS$	-6.98	-7.47	-7.05	-5.98	-4.15
$\%\Delta W$	-2.47	-2.35	-1.90	-1.28	-0.38

Table 11 Quality-enhancing investments model with 4 firms and no efficiencies.

	$\tau = 1.5$				
ρ	0.2	0.3	0.4	0.5	0.6
r	**0.14**	0.15	0.17	0.19	0.21
$\%\Delta pI$	6.49	9.14	11.50	13.59	15.42
$\%\Delta pO$	0.93	1.80	2.77	3.81	4.88
$\%\Delta uI$	-4.50	-5.99	-7.19	-8.20	-9.10
$\%\Delta uO$	0.74	1.43	2.25	3.16	4.17
$\%\Delta \pi I$	0.71	1.48	2.44	3.53	4.70
$\%\Delta \pi O$	2.15	4.19	6.61	9.32	12.31
$\%\Delta CS$	-5.09	-6.12	-6.59	-6.65	-6.43
$\%\Delta W$	-1.63	-1.82	-1.80	-1.67	-1.46

	$\tau = 0.22$				
ρ	0.2	0.3	0.4	0.5	0.6
r	**0.34**	0.35	0.36	0.37	0.38
$\%\Delta pI$	3.01	4.27	5.40	6.42	7.34
$\%\Delta pO$	0.31	0.61	0.96	1.35	1.76
$\%\Delta uI$	-3.33	-4.61	-5.74	-6.74	-7.65
$\%\Delta uO$	0.36	0.72	1.14	1.62	2.15
$\%\Delta \pi I$	0.24	0.48	0.76	1.08	1.41
$\%\Delta \pi O$	0.92	1.83	2.92	4.16	5.50
$\%\Delta CS$	-3.52	-4.64	-5.47	-6.10	-6.56
$\%\Delta W$	-1.38	-1.76	-2.00	-2.15	-2.24

Table 12. Capacity-sharing model with 4 firms and no efficiencies.

	$\tau = 0.5$				
ρ	0.2	0.3	0.4	0.5	0.6
r	**0.13**	0.15	0.16	0.19	0.23
pc	4.48	3.90	3.32	2.76	2.20
pI	5.09	4.62	4.11	3.57	2.97
pO	4.54	4.02	3.53	3.03	2.51
uc	2.40	2.26	2.15	2.07	2.00
uI	3.03	2.75	2.54	2.38	2.25
uO	2.43	2.34	2.29	2.27	2.29
qc	4.80	4.52	4.31	4.14	4.00
qI	4.54	4.12	3.81	3.57	3.38
qO	4.86	4.67	4.57	4.54	4.57
πc	18.62	15.06	12.00	9.27	6.80
ϕ	39.37	32.38	26.47	21.21	16.30
πO	19.10	16.07	13.51	11.17	8.88
ΔCS	-2.96	-4.54	-5.45	-5.77	-5.52
ΔW	-0.36	-1.27	-1.46	-1.21	-0.73

	$\tau = 0.8$				
ρ	0.2	0.3	0.4	0.5	0.6
r	**0.34**	0.37	0.41	0.48	0.58
pc	5.51	4.73	3.99	3.29	2.61
pI	7.43	6.56	5.76	4.99	4.26
pO	5.30	4.57	3.91	3.25	2.54
uc	4.72	4.39	4.14	3.95	3.79
uI	8.17	7.21	6.56	6.14	5.95
uO	4.54	4.25	4.05	3.91	3.69
qc	5.91	5.49	5.18	4.93	4.74
qI	6.64	5.86	5.33	4.99	4.84
qO	5.68	5.31	5.07	4.88	4.61
πc	21.39	16.32	12.10	8.44	5.17
ϕ	55.32	43.07	33.41	25.27	18.15
πO	19.78	15.27	11.60	8.26	4.89
ΔCS	10.84	5.02	1.88	0.55	0.71
ΔW	21.77	14.39	10.58	8.76	8.24

Table 13. Quality-enhancing investments model with 3 firms and efficiencies ($d = 0.25$).

	$\tau = 1.5$				
ρ	0.2	0.3	0.4	0.5	0.6
r	**0.13**	0.14	0.16	0.17	0.19
pc	4.26	3.90	3.54	3.19	2.84
pI	4.57	4.33	4.07	3.78	3.47
pO	4.29	3.97	3.65	3.34	3.03
uc	1.93	1.86	1.81	1.76	1.72
uI	2.05	1.94	1.85	1.77	1.71
uO	1.95	1.89	1.86	1.84	1.83
qc	3.29	3.12	2.98	2.86	2.77
qI	3.12	2.87	2.51	2.27	2.07
qO	3.32	3.19	2.76	2.56	2.40
πc	12.14	10.41	8.91	7.58	6.37
ϕ	25.41	22.01	19.11	16.57	14.26
πO	12.36	10.86	9.60	8.50	7.49
ΔCS	-1.92	-2.68	-3.21	-3.56	-3.73
ΔW	-0.59	-1.03	-1.22	-1.22	-1.10

	$\tau = 0.22$				
ρ	0.2	0.3	0.4	0.5	0.6
r	**0.34**	0.35	0.35	0.36	0.37
pc	4.67	4.52	4.37	4.23	4.09
pI	4.83	4.74	4.64	4.55	4.46
pO	4.64	4.48	4.34	4.20	4.07
uc	2.21	2.15	2.09	2.04	1.99
uI	2.72	2.60	2.49	2.40	2.32
uO	2.19	2.13	2.08	2.03	2.00
qc	1.54	1.47	1.42	1.37	1.32
qI	1.70	1.58	1.51	1.43	1.36
qO	1.52	1.46	1.35	1.28	1.21
πc	4.76	4.36	4.01	3.69	3.41
ϕ	11.56	10.60	9.78	9.05	8.40
πO	4.66	4.27	3.93	3.64	3.38
ΔCS	0.55	0.23	-0.02	-0.23	-0.40
ΔW	2.49	2.02	1.66	1.38	1.16

Table 14. Capacity-sharing model with 3 firms and efficiencies ($d = 0.25$).

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