Online Appendix

We follow Bourreau et al. (2018) to extend the framework to uncertain demand. Formally, the demand level δ is uncertain ex-ante and it is assumed to be common to all local markets and uniformly distributed over $[1-\sigma, 1+\sigma]$, being $\sigma \in (0,1)$ the degree of demand uncertainty.[1] The uncertainty is resolved between the (co-)investment decisions and the request for access, thus access provides a wait-and-see option to the entrant. The timing is:

1. Being the demand level δ unknown, the incumbent chooses the coverage z_1 and the incurred cost in each covered area.
2. Being the demand level δ unknown, the entrant decides upon where to co-invest among the covered areas.
3. The demand level δ realizes.
4. Being the demand level δ known, the entrant decides upon access request among the covered areas where it did not co-invest.

Notice that our analysis departs from Bourreau et al.(2018) in stage one. Unlike their work we take into account the possibility that the incumbent may prefer to over-invest in order to deter co-investment.

In duopoly areas the realized net profit is $\delta\hat{\pi}_i^d(a) - f$ for $i = 1, e$, where f is the interconnection (fixed) cost firms incur when they compete in a local market. Ex-ante, the expected profit is $\hat{\pi}_i^d(a) - f \equiv \pi_i^d(a)$ for $i = 1, e$, where $\pi_i^d(a)$ is the profit in the baseline model. Finally, $\hat{\pi}^d \equiv \hat{\pi}_1^d(0) = \hat{\pi}_e^d(0)$ and $\pi^d \equiv \hat{\pi}^d - f$.

The remaining assumptions are summarized below:

Assumption 2 *(i) There is a maximum access price, a^{max}, above which the entrant incur losses; (ii) $\pi_1^d(a)$ (resp. $\pi_e^d(a)$) is increasing (decreasing) with a for $a < a^{max}$; (iii) if $a \geq a^{max}$, the market is foreclosed and the incumbent makes π^m; (iv) $\lim_{a \to a^{max}} \pi_1^d(a) \leq \pi^m$; (v) co-investment is profitable in expectation, so $\hat{\pi}^d > f$ holds; (vi) it is not possible to commit to access ex-ante, thus $\hat{\pi}_e^d(a) < f$; (vii) access is profitable in the high states of the demand, and consequently $(1+\sigma)\hat{\pi}_e^d(a) > f$.*[2]

Notice that nothing changes in the case of pure co-investment, as uncertainty only resolves after (co-)investment decisions are made. This implies that the analysis without uncertainty extends directly to demand uncertainty in the case of pure co-investment. Therefore, next we proceed to solve the game of co-investment with access. We proceed by backward induction.

[1] As Bourreau et al. (2018) do, we also abstract from other factors that might affect investment under uncertainty, such as risk aversion.

[2] Since $\hat{\pi}_e^d(a) < f < \hat{\pi}^d$, a must be above a minimum threshold, $a^{min} > 0$.

Entrant's access decision: Once the demand is realized, the entrant requests access iff $\delta\hat{\pi}_e^d(a) - f \geq 0$ (equivalently, iff the realization of the demand is large enough: $\delta \geq f/\hat{\pi}_e^d(a) \equiv \underline{\delta}(a)$).[3] Therefore, given the incumbent's coverage z_1 and the co-investment decision of the entrant z_e, if $z_e = z_1$ then there is no access, and if $z_e < z_1$ then the entrant requests access in areas $(z_e, z_1]$ iff $\delta \geq \underline{\delta}(a)$.

Entrant's co-investment decision: Given the incumbent's coverage z_1 and the cost function $c(z)$, the entrant has two choices in each area $z \in [0, z_1]$: co-invest and attain the expected profit $\mathbb{E}(\pi_e^c) \equiv \pi^d - c(z)/2$, or wait and see and attain the expected profit $\mathbb{E}(\pi_e^a) \equiv \int_{\underline{\delta}(a)}^{1+\sigma} \frac{\delta\hat{\pi}_e^d(a)-f}{2\sigma} d\delta = (1-\omega)\hat{\pi}_e^d(a) - p^e f > 0$ with $\omega = (\underline{\delta}(a)^2 - (1-\sigma)^2)/4\sigma$ and ex-ante probability of entry, $p^e = \int_{\underline{\delta}(a)}^{1+\sigma} \frac{d\delta}{2\sigma} = (1+\sigma-\underline{\delta}(a))/2\sigma$.

From $c(z) = z$, the entrant co-invests in area $z \in [0, z_1]$ iff $\pi^d - z/2 \geq \mathbb{E}(\pi_e^a)$ or, equivalently, iff

$$z \leq \tilde{z}^{ca}(a) \equiv 2[\pi^d - \mathbb{E}(\pi_e^a)].$$

Incumbent's (over-)investment decision: As mentioned above, here our analysis differs from Bourreau et al.(2018) since we allow the incumbent to decide not only upon the coverage but also upon the cost incurred in each covered area. Notice that, when $z_1 < \tilde{z}^{ca}(a)$, the entrant always co-invests. But $\tilde{z}^{ca}(a) < \bar{z}^c$,[4] so the incumbent's profit is increasing on this branch and thus the equilibrium coverage is at least equal to $\tilde{z}^{ca}(a)$.

In any covered area z, the incumbent's gain from preventing co-investment is

$$\underbrace{\int_{1-\sigma}^{\underline{\delta}(a)} \frac{\delta\pi^m}{2\sigma} d\delta}_{\omega\pi^m} + \underbrace{\int_{\underline{\delta}(a)}^{1+\sigma} \frac{\delta\hat{\pi}_1^d(a) - f}{2\sigma} d\delta}_{(1-\omega)\hat{\pi}_1^d(a) - p^e f} - \underbrace{\int_{1-\sigma}^{1+\sigma} \frac{\delta\hat{\pi}^d - f}{2\sigma} d\delta}_{\hat{\pi}^d - f} \equiv \mathbb{E}(\pi_1^a) - \pi^d.$$

Since the entrant covers half of the cost incurred by the incumbent, co-investment is deterred iff $\pi^d - (c(z) + \varepsilon_z)/2 \leq \mathbb{E}(\pi_e^a) \Leftrightarrow \varepsilon_z \geq 2[\pi^d - \mathbb{E}(\pi_e^a)] - c(z)$. Thus,

$$\varepsilon_z = 2[\pi^d - \mathbb{E}(\pi_e^a)] - c(z).$$

If co-investment is deterred, the incumbent must bear the cost covered by the entrant, which is equal to $c(z)/2$. Thus, the cost of blocking co-investment in area z is $\varepsilon_z + c(z)/2 = 2[\pi^d - \mathbb{E}(\pi_e^a)] - c(z)/2$.

Therefore, the gain from preventing co-investment is greater than the corresponding cost in a given area z iff

$$\mathbb{E}(\pi_1^a) - \pi^d \geq 2[\pi^d - \mathbb{E}(\pi_e^a)] - c(z)/2.$$

[3] The assumption $(1+\sigma)\hat{\pi}_e^d(a) > f$ can then be rewritten as $\underline{\delta}(a) < 1 + \sigma$.

[4] Pure co-investment is unaffected by uncertainty, so $2[\pi^d - \mathbb{E}(\pi_e^a)] < 2\pi^d$.

- *Complete co-investment deterrence* ($\mathbb{E}(\pi_1^a) \geq 3\pi^d - 2\mathbb{E}(\pi_e^a)$). As in the absence of uncertainty, if the incumbent finds it optimal to deter co-investment in the area with the lowest deployment cost $z = 0$, then it finds it optimal to completely deter co-investment in all areas $z \leq \tilde{z}^{ca}(a)$. To prevent co-investment at $z = 0$, the incumbent over-invests $\varepsilon_0 = \tilde{z}^{ca}(a) \equiv 2[\pi^d - \mathbb{E}(\pi_e^a)]$. Thus, complete deterrence occurs iff $\mathbb{E}(\pi_1^a) \geq 3\pi^d - 2\mathbb{E}(\pi_e^a)$.
  In this case, the incumbent's expected profit is

$$\mathbb{E}(\Pi_1) = z_1\mathbb{E}(\pi_1^a) - \int_0^{z_1} c(z)dz - \int_0^{\tilde{z}^{ca}(a)} \left(2[\pi^d - \mathbb{E}(\pi_e^a)] - c(z)\right) dz.$$

  While the co-investment coverage is 0, from $c(z) = z$ and the FOC with respect to z_1 we determine the equilibrium coverage $z_1 = \tilde{z}^a(a) \equiv \mathbb{E}(\pi_1^a) > \tilde{z}^{ca}(a)$.

- *Partial co-investment deterrence* ($3\pi^d - 2\mathbb{E}(\pi_e^a) > \mathbb{E}(\pi_1^a) > 2\pi^d - \mathbb{E}(\pi_e^a)$). Let us denote the area where co-investment deterrence starts by $\tilde{z}_o \in (0, \tilde{z}^{ca}(a))$. The incumbent's expected profit is

$$\begin{aligned}\mathbb{E}(\Pi_1) &= \tilde{z}_o\pi^d + (z_1 - \tilde{z}_o)\mathbb{E}(\pi_1^a) - \int_0^{\tilde{z}_o} \frac{c(z)}{2}dz - (\tilde{z}^{ca}(a) - \tilde{z}_o)2[\pi^d - \mathbb{E}(\pi_e^a)] \\ &\quad - \int_{\tilde{z}^{ca}(a)}^{z_1} c(z)dz,\end{aligned}$$

  with $z_1 > \tilde{z}^{ca}(a)$. The incumbent maximizes $\mathbb{E}(\Pi_1)$ with respect to $\tilde{z}_o$ and z_1. From $c(z) = z$, we have that $\partial\mathbb{E}(\Pi_1)/\partial\tilde{z}_o = \pi^d - \mathbb{E}(\pi_1^a) - \tilde{z}_o/2 + 2[\pi^d - \mathbb{E}(\pi_e^a)]$ is strictly positive at $\tilde{z}_o = 0$ iff $3\pi^d - 2\mathbb{E}(\pi_e^a) > \mathbb{E}(\pi_1^a)$, thus the incumbent allows for co-investment in low cost areas. From the FOC with respect to $\tilde{z}_o$ we find that

$$\tilde{z}_o = 2[3\pi^d - \mathbb{E}(\pi_1^a) - 2\mathbb{E}(\pi_e^a)],$$

  while the FOC with respect to z_1 yields the equilibrium coverage $z_1 = \tilde{z}^a(a) \equiv \mathbb{E}(\pi_1^a)$. Consequently, co-investment is deterred in all areas $z \in [\tilde{z}_o, \tilde{z}^{ca}(a)]$. Notice that $\tilde{z}_o < \tilde{z}^{ca}(a) < \tilde{z}^a(a)$ since $2\pi^d - \mathbb{E}(\pi_e^a) < \mathbb{E}(\pi_1^a)$.

- *No deterrence* ($\mathbb{E}(\pi_1^a) \leq 2\pi^d - \mathbb{E}(\pi_e^a)$). There is no over-investment when $\tilde{z}_o \geq \tilde{z}^{ca}(a)$, or equivalently, when $2\pi^d \geq \mathbb{E}(\pi_1^a) + \mathbb{E}(\pi_e^a)$. The incumbent's expected profit is

$$\mathbb{E}(\Pi_1) = \tilde{z}^{ca}(a)\pi^d + (z_1 - \tilde{z}^{ca}(a))\mathbb{E}(\pi_1^a) - \int_0^{\tilde{z}^{ca}(a)} \frac{c(z)}{2}dz - \int_{\tilde{z}^{ca}(a)}^{z_1} c(z)dz.$$

  The FOC with respect to z_1 being $c(z) = z$ yields the interior solution $z_1 = \tilde{z}^a(a) \equiv \mathbb{E}(\pi_1^a)$, which is larger than $\tilde{z}^{ca}(a)$ within this region iff $2\pi^d - \mathbb{E}(\pi_e^a) \geq \mathbb{E}(\pi_1^a) > 2\pi^d - 2\mathbb{E}(\pi_e^a)$. The equilibrium coverage is at the corner solution $\tilde{z}^{ca}(a)$ within this region iff $2\pi^d - 2\mathbb{E}(\pi_e^a) \geq \mathbb{E}(\pi_1^a)$.

The proposition below summarizes these findings.

Proposition 2A *Let the level of demand be uncertain, then with access obligations and co-investment we have that:*

- *The incumbent over-invests to prevent any co-investment iff* $3\pi^d - 2\mathbb{E}(\pi_e^a) \leq \mathbb{E}(\pi_1^a)$ *(complete co-investment deterrence).*
- *The incumbent over-invests to prevent co-investment in the intermediate cost areas* $z \in [\tilde{z}_o, \tilde{z}^{ca}(a)]$*, and allows for co-investment in the low cost areas* $z < \tilde{z}_o$ *iff* $3\pi^d - 2\mathbb{E}(\pi_e^a) > \mathbb{E}(\pi_1^a) > 2\pi^d - \mathbb{E}(\pi_e^a)$ *(partial co-investment deterrence).*
- *There is no over-investment whatsoever iff* $\mathbb{E}(\pi_1^a) \leq 2\pi^d - \mathbb{E}(\pi_e^a)$ *(no co-investment deterrence).*

The local expected profit of the incumbent under access once the coverage cost is paid, $\mathbb{E}(\pi_1^a)$, plays a key role in the previous proposition, so it is useful to look at its shape more closely. Recall that $\mathbb{E}(\pi_1^a) = \omega\pi^m + (1-\omega)\hat{\pi}_1^d(a) - p^e f$, and notice that ω increases with a. Then, when the access fee increases, the increase in the expected monopoly profit and the decrease in the expected coordination costs paid push $\mathbb{E}(\pi_1^a)$ up. The variation in the expected gross duopoly profit with access can be either positive or negative: $\hat{\pi}_1^d(a)$ increases with the access fee but $1-\omega$ decreases. Thus, $\mathbb{E}(\pi_1^a)$ decreases with the access fee when the latter change is negative and it more than offset the former two.

The numerical simulations below show that the incumbent still has incentives to over-invest in the presence of uncertainty (that is, that there are parameter constellations such that the total and partial deterrence regions are not empty).

As we did in Section 5, we fix $\alpha = \beta = 1$, $f = 0.12$, and $\gamma = 0.25$. Figure 4 plots the regions of Proposition 2A for the cases $(a)\,\sigma = 0.15$, where $a^{max} = 0.24318$ and $a^{min} = 0.19564$; and $(b)\,\sigma = 0.3$, where $a^{max} = 0.2822$ and $a^{min} = 0.19564$. In the first case, the incumbent finds it profitable to partially deter co-investment for feasible values of the access fee below 0.2154, whereas complete deterrence is optimal for feasible values of the access fee above the aforesaid threshold.[5] In the second case, the threshold between partial and total deterrence is $a = 0.2127$.[6]

[5] If we take for instance $a = 0.2$, then $z_1 = \tilde{z}^a(a) = 0.20913$, $\tilde{z}^{ca}(a) = 0.14437$ and $\tilde{z}_o = 0.0223$, so co-investment is deterred in areas $z \in [0.0223, 0.14437]$.

[6] If we take again $a = 0.2$, then $z_1 = \tilde{z}^a(a) = 0.2083$, $\tilde{z}^{ca}(a) = 0.1355$ and $\tilde{z}_o = 0.0063$, so co-investment is deterred in areas $z \in [0.0063, 0.1355]$.

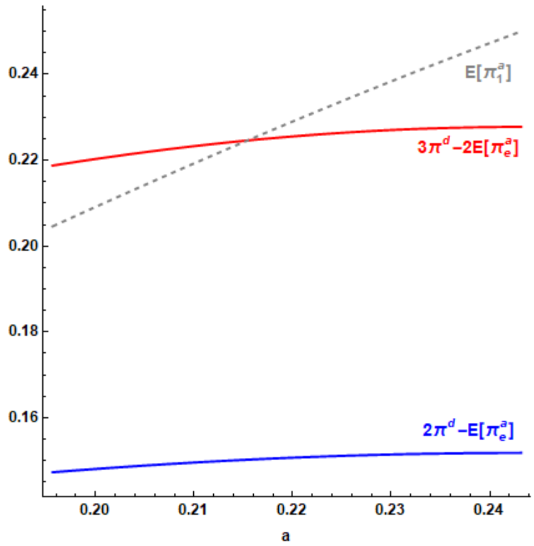

Fig. 4a. $\gamma = 0.25$, $\sigma = 0.15$.

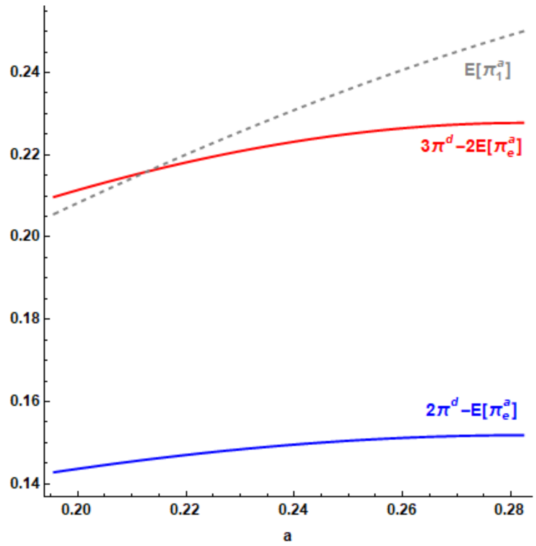

Fig. 4b. $\gamma = 0.25$, $\sigma = 0.30$.

Intuitively, $\gamma = 0.25$ can be seen as a moderate degree of substitutability between products. In this environment of not-so-low competition, the incumbent has a strong incentive to deter co-investment. Notice that $\mathbb{E}(\pi_1^a)$ is increasing because, although $\hat{\pi}_1^d(a) = \pi_1^d(a) + f > \pi^m$ for all the values of the access fee in the support, the difference is not large enough. When we increase the uncertainty level from 0.15 to 0.3, the support of values of the access fee also increases, $\mathbb{E}(\pi_1^a)$ flattens, and the curves $3\pi^d - 2\mathbb{E}(\pi_e^a)$ and $2\pi^d - \mathbb{E}(\pi_e^a)$ shift down, resulting in a larger proportion of the support where complete deterrence is predicted.

Figure 5 plots the regions of Proposition 2A assuming $\alpha = \beta = 1$, $f = 0.10$ and $\gamma = 0.10$. Here, the value of f is slightly adapted so that Assumption 2 is satisfied. As above, we represent the cases for $(a)\, \sigma = 0.15$, where $a^{max} = 0.363$ and $a^{min} = 0.32$; and $(b)\, \sigma = 0.3$, where $a^{max} = 0.398$ and $a^{min} = 0.32$. In the first case, the incumbent never finds it profitable to deter co-investment for any feasible value of the access fee.[7] In the second case, the incumbent finds it profitable to partially deter co-investment for feasible values of the access fee below 0.379, while no deterrence is optimal for feasible values of the access fee above that threshold.[8]

[7]If we take for instance $a = 0.35$, then $z_1 = \tilde{z}^{ca}(a) = 0.252$ and $\tilde{z}_o = 0.257$, so clearly co-investment is not deterred.

[8]If we take again $a = 0.35$, then $z_1 = \tilde{z}^{a}(a) = 0.253$, $\tilde{z}^{ca}(a) = 0.247$ and $\tilde{z}_o = 0.242$, so co-investment is deterred in areas $z \in [0.242, 0.247]$.

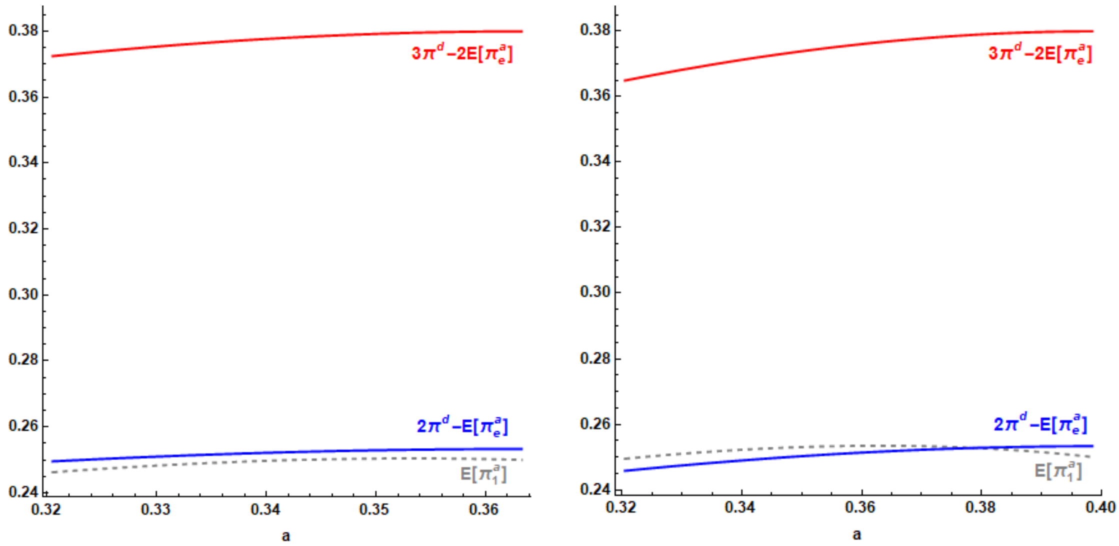

Fig. 5a. $\gamma = 0.10$, $\sigma = 0.15$. Fig. 5b. $\gamma = 0.10$, $\sigma = 0.30$.

Notice that $\gamma = 0.10$ represents a decrease in the degree of substitutability with respect to the previous simulation, and thus the attractiveness of deterring co-investment gets reduced since the business stealing effect is softened. $\mathbb{E}(\pi_1^a)$ has an inverted-U shape since, for high values of the fee in the support, $\hat{\pi}_1^d(a) = \pi_1^d(a) + f$ is significantly larger than π^m. When we increase the uncertainty level from 0.15 to 0.3, we pass from a situation where co-investment is never deterred to a situation where co-investment is partially deterred only for low values of the access fee.