Mobile termination rates and retail regimes in Europe and the US: a unified theory of CPP and RPP*

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Abstract

We analyze an oligopoly model where mobile operators may charge subscribers for placing and receiving calls. We compare the CPP equilibrium (where receiving calls is free) with the RPP equilibrium (where placing and receiving calls are priced equally). Reducing termination rates leads to lower prices and higher penetration under CPP, but has reversed effects under RPP. No termination rate yields efficiency under either retail regime. For intermediate values of call externality, CPP with termination regulated at cost (EU practice) yields higher producer and consumer surplus than RPP with Bill and Keep (US practice). The US scenario is better for consumers (producers) when call externality is higher (lower).

Key words: CPP, RPP, interconnection, call externality, regulation.

JEL Classification: D43, L13, L51.

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1 Introduction

The liberalization of telecommunication markets has raised the issue of interconnection between competing mobile networks: How to price the wholesale service of terminating a call originated on a rival network? This question has been debated over the last two decades by regulators, firms and economists all over the world but has not lead to a unique approach.

In most countries, including those pertaining to the EU, regulators set the mobile termination rate to be paid by the originating operator, allowing the terminating operator to recover costs. In other countries, instead, a 'Bill and Keep' termination regime applies: no termination rates are paid.[1] Which approach is better? This paper contributes to the theory on the impact of termination rates on retail pricing, usage, market penetration and welfare with a formal analysis. This allows us to look back at the history of the mobile termination rate debate, and assess the validness of the arguments put forward by market players and academics.

One argument in favour of Bill and Keep is that firms should recover their termination costs by charging subscribers for receiving calls. Competition would discipline those reception charges and lead to more efficient outcomes. Receiving-Party-Pays (RPP) regimes have not been observed in Europe, but they are common in the US, Canada, Hong Kong and Singapore, proving that receiving calls is valuable. These receiver benefits, or call externalities, have been ignored in most of the literature, focussing on the Calling-Party-Pays (CPP) regime where firms only charge for placing calls.[2] Instead, we build upon Jeon et al. (2004) (hereafter JLT) who extend the traditional symmetric model of network competition with price discrimination between on- and off-net traffic to allow for call externalities and reception charges. However, our model and analysis exhibit three important differences.

First, JLT consider a duopoly while we allow for any number of firms. This proves to be crucial for the existence of equilibria without 'connectivity breakdown' (see also Hoernig, 2016). Second, because the model admits a continuum of equilibria,[3] JLT restrict attention to the unique equilibrium that is robust to adding vanishing noise to the receiver's utility.[4] This equilibrium admits strictly positive but different prices for placing and receiving calls. Instead, we focus on two practically relevant equilibria: the CPP equilibrium, where reception is not charged (as in Europe), and the RPP equilibrium,

[1] While in Canada, Hong Kong and Singapore Bill and Keep was imposed by the regulator, in the US it resulted from free bilateral negotiations on reciprocal termination rates (Ofcom, 2009, Annex 8.1)

[2] See Armstrong (2002), Vogelsang (2003) and Peitz et al. (2004) for extensive reviews of the CPP literature on termination charges and network competition that ignores call externalities. Harbord and Pagnozzi (2010) emphasize the importance of call externalities, even when restricted to CPP regimes.

[3] See Hurkens and López (2014b) for a complete analysis.

[4] Cambini and Valletti (2008) and Hoernig (2016) also use this approach. López (2011), instead, selects a unique equilibrium by adding non-vanishing noise.

where placing and receiving calls are priced equally (as in the US). Third, we assume passive rather than responsive expectations. This assumption explains why the incentives of profit- and welfare-maximizers (*i.e.*, firms and regulators) are misaligned, consistent with the observed opposition by large networks against termination rate reductions in the EU (see Hurkens and López, 2014a).

We first provide a full characterization of equilibrium prices and profit when subscription demand is inelastic. In line with previous literature, on-net prices efficiently internalize the call externality whereas off-net prices are set above perceived marginal cost to make rival networks relatively unattractive. Call volume is decided by the caller in CPP and by the receiver in RPP. Off-net prices are therefore increasing in termination rate under CPP and decreasing under RPP. It follows that firms prefer inefficiently high (respectively, low) termination rates under CPP (respectively, RPP). However, no termination rate induces efficient off-net call volume: For sufficiently low termination rates the CPP equilibrium breaks down because firms then charge reception; for sufficiently high termination rates the RPP equilibrium breaks down because firms then raise the call price. Our results extend to the case of elastic subscription demand. Incentives to raise off-net prices above perceived marginal cost persist but are less strong because of the resulting reduction in overall penetration. Lowering (respectively, raising) termination rates increases penetration under CPP (respectively, RPP). This indicates that the so called 'externality surcharge', applied in some European countries, was welfare-reducing and profit-increasing (see Ofcom, 2007, page 147).

Having established the impossibility to regulate termination rates efficiently, we proceed to compare two relevant sub-optimal options: we compare the CPP equilibrium when termination rates are regulated at cost (as recommended in the EU since 2009) with the RPP equilibrium when termination rates are set at zero (as observed in the US and Canada). This allows us to obtain theoretical predictions on the relative performance of European and American markets in terms of consumer, producer and total welfare. For relatively low call externality, consumer and total surplus is higher under CPP; for relatively high call externality, consumer surplus and penetration are lower and profits higher under CPP. This can be contrasted with the evidence from a few empirical studies that call volume is higher but penetration lower in RPP countries.[5]

The rest of the paper is organised as follows. Section 2 introduces the model. Section 3 derives the equilibria for the CPP and RPP regimes. Section 4 compares the outcomes obtained under the CPP regime with cost-based termination rates with those obtained under the RPP regime with Bill and Keep. Section 5 extends the model to allow for elastic participation. Section 6 concludes by reviewing the historical debate on termination rates in the light of our results. Proofs are relegated to the Appendix.

[5] See Littlechild (2006), Dewenter and Kruse (2011) and Ofcom (2009, Annex 7).

2 The model

We consider a general model of $n \geq 2$ network operators. The n network operators have complete coverage and compete for a continuum of consumers of unit mass.

Timing. We assume that the terms of interconnection are negotiated or established by a regulator first. Then, for a given reciprocal[6] access charge $a \geq 0$, the timing of the game is the following:

1. Consumers form expectation α_i^e about the number of subscribers of network i.
2. Firms take these expectations as given and choose simultaneously retail tariffs.
3. Consumers make rational subscription and consumption decisions, given their expectations and given the networks' tariffs.

Consumers thus hold passive expectations which do not change when a firm changes a price. Realized market share α_i is a function of prices and consumer expectations. Self-fulfilling expectations imply that at a symmetric equilibrium $\alpha_i^e = \alpha_i = 1/n$.[7]

Cost structure. The fixed cost to serve each subscriber is f, whereas c_O and c_T denote the marginal cost of providing a telephone call borne by the originating and terminating networks. The marginal cost of an on-net call is $c = c_O + c_T$. Let us denote the termination mark-up by

$$m = a - c_T.$$

The perceived marginal cost of an off-net call for the originating network is the true cost c for on-net calls, augmented by the termination mark-up for the off-net calls: $c_O + a = c + m$. The perceived marginal cost of an off-net call for the terminating network is $c_T - a = -m$.

Pricing. Each firm $i \in N = \{1, 2, \ldots, n\}$ charges a tariff $T_i = (F_i, p_i, r_i, \hat{p}_i, \hat{r}_i)$, consisting of a fixed fee (F_i), per-unit call and reception charges for on-net traffic (p_i and r_i) and per-unit call and reception charges for off-net traffic ($\hat{p}_i$ and $\hat{r}_i$).[8] We restrict all prices to be non-negative.[9]

Individual demand. Subscribers obtain positive utility from making and receiving calls. The caller's utility from making a call of length q minutes is $u(q)$, whereas the

[6]Reciprocity means that a network pays as much for termination of a call on the rival network as it receives for completing a call originated on the rival network.

[7]If consumers form expectations after tariffs are observed (as assumed in JLT), consumers' expectations are responsive because they would depend on the prices chosen. See Hurkens and López (2014a) for an extensive discussion of passive and responsive expectations.

[8]This is w.l.o.g. given our restriction to reciprocal termination rates and symmetric equilibria, and reduces burden of notation.

[9]In practice, it has been very rare for firms to pay for receiving calls (see López, 2011, section 5).

receiver's is $\widetilde{u}(q)$ from receiving a call of that length. $u(\cdot)$ and $\widetilde{u}(\cdot)$ are twice continuously differentiable, increasing and concave. For tractability, we assume that

$$\widetilde{u}(q) = \beta u(q) \quad \text{with } 0 < \beta < 1,$$

where β measures the strength of the call externality. The caller's demand function is given by $u'(q(p)) = p$, whereas the receiver's demand function is given by $\widetilde{u}'(q(r)) = r$. We consider the case in which both callers and receivers can hang up. Defining $D(p,r) = q(\max\{p, r/\beta\})$, the length of an on-net call is $D(p_i, r_i)$, whereas the length of an off-net call is $D(\hat{p}_i, \hat{r}_j)$ (for $i \in N$ and $j \in N \setminus \{i\}$). We will denote $U(p,r) = u(D(p,r))$ and $\widetilde{U}(p,r) = \beta u(D(p,r))$.

Market shares. We are interested in allowing for industry structure with more than two firms. We will use the Logit formulation.[10] We make the standard assumption of a balanced calling pattern, which means that the fraction of calls from a given subscriber of a given network and completed on another given (including the same) network is equal to the fraction of consumers subscribing to the terminating network.[11] Let w_i denote the expected value of subscribing to network i. That is,

$$\begin{aligned} w_i \;=\; & \alpha_i^e(U(p_i,r_i) + \widetilde{U}(p_i,r_i) - (p_i + r_i)D(p_i,r_i)) - F_i \\ & + \sum_{j \neq i} \alpha_j^e(U(\hat{p}_i,\hat{r}_j) - \hat{p}_i D(\hat{p}_i,\hat{r}_j)) \\ & + \sum_{j \neq i} \alpha_j^e(\widetilde{U}(\hat{p}_j,\hat{r}_i) - \hat{r}_i D(\hat{p}_j,\hat{r}_i)). \end{aligned} \tag{1}$$

The first line corresponds to the utility from placing and receiving on-net calls, the second to the utility from placing off-net calls and the third to the utility from receiving off-net calls. Consumers have idiosyncratic tastes for each operator. We add a random noise term ε_i and define $W_i = w_i + \mu\varepsilon_i$. The parameter $\mu > 0$ reflects the degree of product differentiation in a Logit model. A high value of μ implies that most of the value is determined by a random draw so that competition between the firms is rather weak. The noise terms ε_k are random variables of zero mean and variance $\pi^2/6$, identically and independently double exponentially distributed. These terms reflect consumers' preference for one good over another (they are known to the consumer but are unobserved by the firms). A consumer will subscribe to network $i \in N$ if and only if $W_i > W_j$ for all $j \in N \setminus \{i\}$. The probability of subscribing to network i equals i's market share:

$$\alpha_i = \frac{\exp[w_i/\mu]}{\sum_{k=1}^{n} \exp[w_k/\mu]}. \tag{2}$$

[10] See Anderson and De Palma (1992) and Anderson et al. (1992) for details about the Logit model.

[11] Dessein (2003, 2004) examines how unbalanced calling patterns between different customer types affect retail competition when network operators compete in the presence of the caller-pays regime.

Consumer Surplus. Consumer surplus in the Logit model has been derived by Small and Rosen (1981) as (up to a constant)

$$CS = \mu \ln \left(\sum_{k=1}^{n} \exp(w_k/\mu) \right) = w + \mu \ln n, \tag{3}$$

where the last equation holds in case of a symmetric solution where each network offers surplus $w_i = w$.

Profit. Fixing tariffs of firms $j \neq i$ at $(F^*, p^*, r^*, \hat{p}^*, \hat{r}^*)$, firm i's profit is given by

$$\begin{aligned} \pi_i &= \alpha_i \Big[\alpha_i (p_i + r_i - c) D(p_i, r_i) + F_i - f \Big] + \\ & \alpha_i \Big[(1 - \alpha_i)(\hat{p}_i - c - m) D(\hat{p}_i, \hat{r}^*) + (1 - \alpha_i)(\hat{r}_i + m) D(\hat{p}^*, \hat{r}_i) \Big]. \end{aligned} \tag{4}$$

3 Equilibrium analysis

The game specified in the previous section admits many equilibria. First, there always exist bad "mis-coordination" equilibria without any off-net traffic, in which all firms charge infinite call and reception prices for off-net traffic.[12] Second, there often exists a plethora of symmetric equilibria without connectivity breakdown (that is, with strictly positive off-net call volume). Hurkens and López (2014b) characterize the full set of such equilibria. This indeterminacy of equilibria (first observed by JLT, p. 89) originates from the assumption that call volume $D(p, r)$ depends only on the call price when $p > r/\beta$ and only on the receiver charge when $p < r/\beta$. So only one of these prices can be pinned down by the relevant first-order condition. This holds both for on-net and off-net prices.

On-net prices. The multiplicity of equilibrium on-net call and reception charges is innocuous. The reason is that in equilibrium firms set those charges so as to maximize the utility obtained from on-net traffic by internalizing the call externality. Optimality requires that the volume of on-net traffic q satisfies $(1+\beta)u'(q) = c$. This can be obtained by setting prices (p^*, r^*) where

$$p^* = \frac{c}{1+\beta}, \quad r^* = \frac{\beta c}{1+\beta}, \tag{5}$$

but the optimal volume is also obtained when setting on-net prices (p^*, r_i) with $r_i < r^*$ or (p_i, r^*) with $p_i < p^*$.

Off-net prices. The multiplicity of off-net call and reception charges is more involved and translates into multiple equilibrium off-net call volumes. This creates a coordination (or equilibrium selection) problem. The optimal off-net call price $\hat{p} > \hat{r}/\beta$ depends on

[12] On-net prices will be set efficiently and fixed fees are used to fight for market share. Firms will not make any profit on call and reception services and just choose fixed fee to maximize $\pi = \alpha(F - f)$. In a symmetric equilibrium this yields $F^* = f + n\mu/(n-1)$ and profit $\pi^* = \mu/(n-1)$.

the rival's off-net reception charge $\hat{r}$: an increase in $\hat{p}$ reduces call volume and negatively affects subscribers on rival networks as their marginal utility for receiving calls exceeds the price $\hat{r}$ they pay for it. The size of this pecuniary externality depends on that reception charge. A similar argument applies to equilibria with $\hat{r}/\beta > \hat{p}$ where call volume is determined by the receiver.

JLT, Cambini and Valletti (2008) and Hoernig (2016) solve the indeterminacy by allowing for vanishing noise in the receiver's utility. This yields, for a given termination rate, a unique equilibrium with positive but different prices for placing and receiving calls. We take a different approach by considering two plausible and practically relevant avenues for solving the coordination problem.

First, we consider the equilibrium with $r = \hat{r} = 0$, that is where subscribers do not pay for receiving calls. We call this the CPP (calling-party-pays) equilibrium. The rationale for this selection is that in European countries, subscribers of mobile (and fixed) telecommunication providers have historically never paid for receiving calls. The characterization of the unique CPP equilibrium is similar to the one established in the previous literature based on duopoly models with call externalities where firms are not allowed to charge for reception (Berger (2005), Hurkens and López (2014a, section 3.4), JLT, Prop. 8). However, since our model does allow for setting reception charges, we must impose additional restrictions so that no firm has an incentive to set a strictly positive reception charge. Interestingly, it turns out that those necessary conditions exclude the existence of efficiently regulated CPP equilibria in the presence of call externality. This is in stark contrast to Berger (2005).

Second, we consider equilibria with $r = p$ and $\hat{r} = \hat{p}$, that is where there is no distinction between the price of placing and receiving calls. This equilibrium selection is motivated by the fact that in the US (and some other countries), subscribers to mobile and fixed telecommunication providers have historically always paid the same price for placing and receiving calls (see Ofcom, 2009, Annex 9, A1.26).[13] We call this the RPP (receiving-party-pays) equilibrium.[14]

3.1 CPP

Here, we characterize the unique symmetric equilibrium in which receiving calls is free of charge: $r = \hat{r} = 0$. As we already argued, on-net call volume will be efficient: this is achieved in a CPP equilibrium by setting $p = p^* = c/(1+\beta)$ and $r = 0$. Since $\hat{r} = 0$ in a CPP equilibrium, callers determine the volume of off-net calls, and the equilibrium

[13] Currently, both in Europe and in the US subscribers often have the option to buy a bundle of limited minutes for a fixed price. Minutes beyond the limit are charged extra. In the US such offers usually count both received and placed calls towards the limit, whereas in Europe only placed calls are counted.

[14] Strictly speaking, any equilibrium with a strictly positive reception charge (such as the one in JLT) could be termed RPP, but we use the term exclusively in case of *equal* call and reception prices.

off-net call price $\hat{p}^*$ is determined by the corresponding first-order condition. We show in the Appendix that this implies

$$\hat{p}^* = \frac{(n-1)(c+m)}{n-1-\beta}. \tag{6}$$

Clearly, the off-net call price increases with the perceived marginal cost of originating an off-net call, $c+m$. Without call-externality ($\beta = 0$) the off-net call price simply equals $c+m$. With call-externality the price is distorted upward to reduce the benefit for receivers on rival networks. This upward distortion is strongest in case of duopoly ($n = 2$). In the extreme case, as $\beta \to 1$, the call price then goes to infinity and there is asymptotic connectivity breakdown (see JLT). However, such breakdown does not occur beyond duopoly as long as $q(2(c+m)) > 0$. Defining $R(p) = (p-c)q(p)$, we characterize the CPP equilibrium in Proposition 1. In particular, we establish necessary conditions for no firm to have an incentive to charge for reception.

Proposition 1. *(CPP)*

[i] A symmetric CPP equilibrium has $r = \hat{r} = 0$, $p = p^* = \frac{c}{1+\beta}$, $\hat{p} = \hat{p}^*$ *and*

$$F^* = f + \frac{n\mu}{n-1} - \frac{2}{n}R(p^*) - \frac{n-2}{n}R(\hat{p}^*). \tag{7}$$

Profit equals

$$\pi^* = \frac{\mu}{n-1} - \frac{1}{n^2}R(p^*) + \frac{1}{n^2}R(\hat{p}^*). \tag{8}$$

[ii] Necessary conditions for the existence of a CPP equilibrium are

$$m \geq \underline{m}^{CPP} \equiv \frac{-\beta c}{\beta + 1 - \frac{\beta}{n-1}}, \tag{NC1}$$

and

$$(1-(n-1)\beta)(u(q(\hat{p}^*)) - \hat{p}^* q(\hat{p}^*)) \leq (n-1)(\beta\hat{p}^* + m)q(\hat{p}^*). \tag{NC2}$$

Proposition 1(i)'s characterization of prices and profit in a CPP equilibrium is a straightforward generalization of Hurkens and López (2014a), where reception charges are ruled out exogenously, to oligopoly and call externalities. The novel conditions in Proposition 1(ii) are required to limit the incentives to charge for reception.

Profit depends on termination charge only through $\hat{p}^*$. If $R(p)$ is quasi-concave and reaches the maximum at monopoly price p^M, equilibrium profit π^* is increasing in m whenever $\hat{p}^* < p^M$. The termination mark-up that maximizes firms' profits is the one that yields $\hat{p}^* = p^M$ and thus equals

$$m^\pi = \frac{n-1-\beta}{n-1}p^M - c,$$

which satisfies (NC1): $m^{\pi} > \underline{m}^{\text{CPP}}$, because $p^M > c$.

The socially optimal termination mark-up would be the one that achieves the efficient call volume, *i.e.* such that $\hat{p}^* = p^*$. Hence, the optimal termination mark-up would be

$$m^W_{\text{CPP}} = \left(\frac{-\beta c}{1+\beta}\right)\left(\frac{n}{n-1}\right) < 0.$$

However, it is easily established that $m^W_{\text{CPP}} < \underline{m}^{\text{CPP}}$, violating (NC1). By Proposition 1(ii), regulating termination charges at efficient levels is incompatible with CPP.[15]

To provide some intuition behind the two necessary conditions, note that neither subscribers' utilities nor firms' profits are affected when network i unilaterally deviates to $\hat{r}_i = \beta\hat{p}^*$ and $F_i = F^* - (1-1/n)\beta\hat{p}^* q(\hat{p}^*)$: off-net call volume is still $q(\hat{p}^*)$ while the additional payment of reception charges is exactly compensated by the reduced fixed fee. We now explain how a network can profitably deviate from this alternative pricing strategy when either of the two conditions is not satisfied.

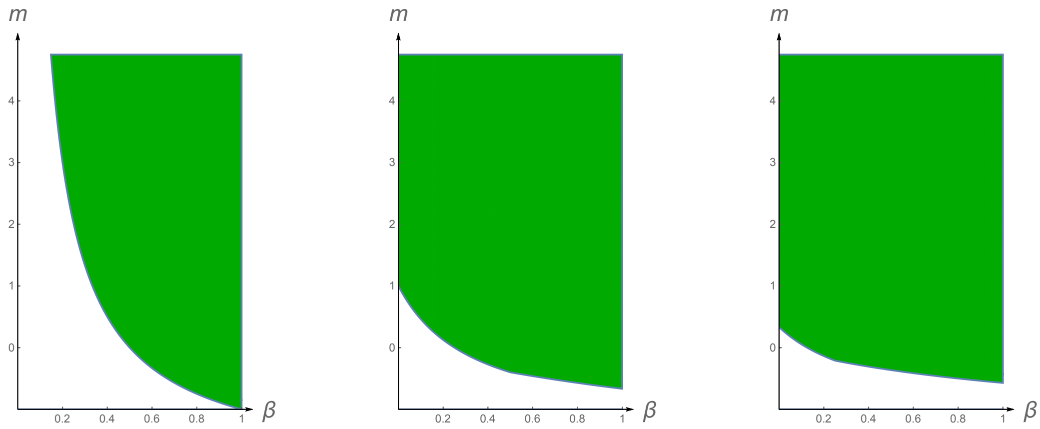
First, suppose that (NC1) is violated (*i.e.*, $m < \underline{m}^{\text{CPP}}$ or, equivalently, $\beta\hat{p}^* < -m$). Now a marginal increase of $\hat{r}_i$ above $\beta\hat{p}^*$ raises i's profit since: (i) it has no effect on subscribers's utilities from off-net calls directed to network i, because both caller and receiver effectively pay a price equal to their marginal utility; and (ii) it reduces the volume of loss-making incoming traffic (as $\beta\hat{p}^* + m < 0$).

Next, suppose (NC1) holds but (NC2) is violated, implying that $(n-1)\beta < 1$. Provoking connectivity breakdown through a high off-net reception charge is profitable, because it hurts own subscribers less than those of rival networks. It makes the own network thus *relatively* more attractive and allows network i to raise the fixed fee by $(1-(n-1)\beta)(u(q(\hat{p}^*)) - \hat{p}^* q(\hat{p}^*))/n$, while keeping market share constant. When (NC2) is violated, this more than off-sets the resulting loss in revenues from incoming off-net traffic (equal to $\beta\hat{p}^* + m \geq 0$ per minute).[16]

Fig. 1 illustrates in (β, m)-space the existence of a CPP equilibrium. Note that the CPP equilibrium may exist even when $m < 0$, that is when the termination rate is below cost. In particular, for sufficiently strong call externality, the CPP equilibrium exists even under Bill and Keep ($m = -c_T$).

[15] If firms are prevented from the option to charge reception (perhaps by law), then efficiency can be achieved in a CPP equilibrium (see Berger, 2005; Hurkens and López, 2014a).

[16] Observe that if $(n-1)\beta > 1$, then (NC1) implies (NC2). Provoking connectivity breakdown now hurts own subscribers more than those of rival networks so that one would need to *reduce* the fixed fee to keep market shares constant, which is clearly unprofitable. Nevertheless, these conditions are not always sufficient for the existence of the CPP equilibrium because provoking connectivity breakdown while inducing different market shares is sometimes optimal. This is the case, for example, when $n = 3$, $q(p) = p^{-2}$, $\mu = 80$, $c = 1$, $\beta = 0.5001$, $m = \underline{m}^{\text{CPP}} + 0.000001$.



(a) $n = 2$ (b) $n = 3$ (c) $n = 5$

Figure 1: CPP existence in (β, m)-space when $c = 1$, $q(p) = p^{-2}$, $\mu = 40$ and $f = 0$.

3.2 RPP

We now characterize the unique symmetric equilibrium in which placing and receiving calls are priced the same: $p = r$ and $\hat{p} = \hat{r}$. In such RPP equilibrium call volume is determined by the receiver. As before, on-net call volume will be efficient and this is achieved in an RPP equilibrium by setting $p = r = r^* = \beta c/(1+\beta)$. Off-net call volume is determined by the first-order condition with respect to $\hat{r}$. We show in the Appendix that this implies

$$\hat{r}^* = -\frac{\beta(n-1)m}{n\beta - 1}. \tag{9}$$

Existence requires $m < 0$ and $n\beta > 1$. The off-net reception charge increases with the cost of receiving an off-net call, $-m$. In other words, a reduction of the termination rate induces higher prices and lower call volume. As $\beta \uparrow 1$, the reception charge converges to $-m = c_T - a$, the perceived marginal cost of terminating a call. If termination charges cannot be negative ($m \geq -c_T$), $\hat{r}^* > c_T$ for all $\beta < 1$. The reception charge for off-net calls is distorted upward to reduce the benefit for callers on rival networks. This distortion is larger when call externality is low. Defining $\tilde{R}(r) = (2r - c)q(r/\beta)$, next we characterize the RPP equilibrium and establish necessary conditions for its existence.

Proposition 2. *(RPP)*

[i] A symmetric RPP equilibrium has $p = r = r^* = \frac{\beta c}{1+\beta}$, $\hat{p} = \hat{r} = \hat{r}^*$ *and*

$$\tilde{F}^* = f + \frac{n\mu}{n-1} - \frac{2}{n}\tilde{R}(r^*) - \frac{n-2}{n}\tilde{R}(\hat{r}^*). \tag{10}$$

Profit equals

$$\tilde{\pi}^* = \frac{\mu}{n-1} - \frac{1}{n^2}\tilde{R}(r^*) + \frac{1}{n^2}\tilde{R}(\hat{r}^*). \tag{11}$$

[ii] Necessary conditions for the existence of an RPP equilibrium are

$$m \leq \overline{m}^{RPP} = \frac{c(1-n\beta)}{n-2+n\beta} \tag{NC3}$$

and

$$(n-1)\beta \geq 1. \tag{NC4}$$

When $(n-1)\beta < 1$ the second-order condition fails to hold, so that firms then would set a high off-net reception charge to create connectivity breakdown. (NC3) is equivalent to $\hat{r}^* \geq \beta(c+m)$ and ensures that a (marginal) deviation by setting $\hat{p}^*$ above $\hat{r}^*/\beta$ is not profitable.[17]

The social welfare maximizing termination mark-up would be the one such that $\hat{r}^* = r^*$, so that $m^W_{\text{RPP}} = c(1-n\beta)/((1+\beta)(n-1))$. However, $m^W_{\text{RPP}} > \overline{m}^{\text{RPP}}$ whenever $\beta < 1$, thereby violating (NC3). By Proposition 2(ii), regulating termination charges at efficient levels is incompatible with an RPP equilibrium. Alternatively, regulators could try to impose the highest termination mark-up compatible with RPP (*i.e.*, $\overline{m}^{\text{RPP}}$), and accept some inefficiency. However, it is in the interest of firms to agree on even lower termination rates (since $\tilde{R}(\hat{r}(\overline{m}^{\text{RPP}})) < 0$), leading to higher prices and lower welfare. In particular, Bill and Keep may maximize profit. Free commercial negotiations on reciprocal termination charges can thus explain Bill and Keep arrangements, as observed in the US, but do not lead to efficient pricing.[18]

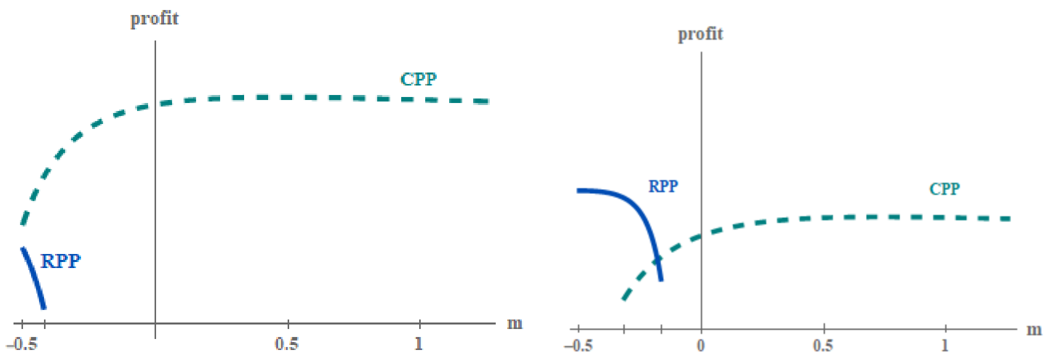
Remark. Note that in the special case where $\beta = 1$, $c_T = c/2$ and $n \geq 3$, Bill and Keep ($m = -c_T$) in fact allows for an RPP equilibrium with $\hat{r}^* = r^* = c/2$. This is fully efficient and there is no price discrimination between on-net and off-net calls.

4 Comparing outcomes in Europe and US

We are now ready to compare historical outcomes in telecommunication markets in Europe and the US in the light of the above analysis of CPP and RPP equilibria. In Europe mobile termination rates were initially unregulated and very high: mobile firms set high termination rates as most incoming calls came from the incumbent monopolist fixed line network. As they made large termination profits, they had no incentive to charge for reception and reduce the volume of incoming calls. When mobile penetration increased, traffic between mobile networks became more relevant, but firms kept playing the CPP equilibrium: as our analysis of competition between mobile networks has shown, RPP

[17]Similar to the case of the CPP equilibrium, even strict versions of (NC3) and (NC4) are not yet sufficient because of the possibilities to profitably deviate by inducing market shares different from $1/n$.

[18]In contrast, Hoernig (2016) and Cambini and Valletti (2008) show that the interests of firms and regulator are perfectly aligned so that free negotiations always lead to efficient pricing. Their results depend on the assumption of responsive expectations (see also Hurkens and López, 2014a,b).



(a) $\beta = 0.8$

(b) $\beta = 0.4$

Figure 2: Profit in CPP and RPP equilibrium ($n = 4$, $c_O = c_T = 0.5$, $q(p) = p^{-2}$)

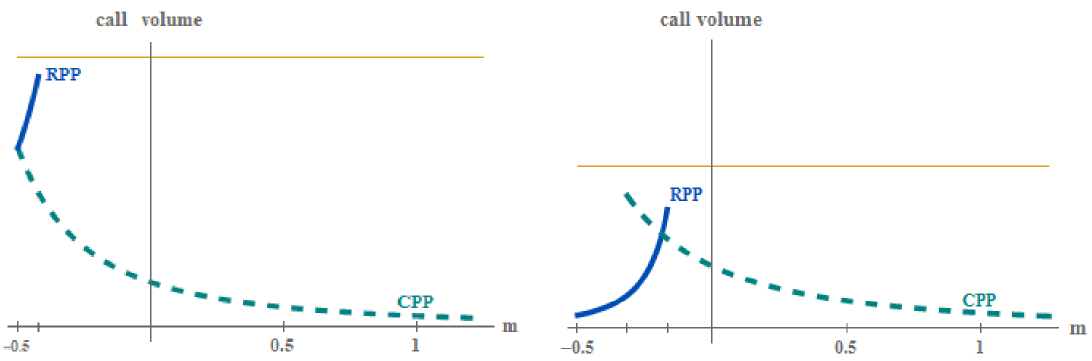
equilibria do not exist for positive termination mark-ups. During two decades, regulators argued and pushed for lower termination charges, while being opposed by the firms. It took about 10 to 20 years, depending on the country, to push termination charges down to cost.

In the US, on the other hand, termination charges were low from the beginning as they had to be reciprocal with respect to incumbent fixed line networks (see Marcus, 2004). When termination rates are below cost, firms have incentives to charge for reception as they can then control better the volume of incoming calls. Indeed, our analysis has shown that for very low termination charges no CPP equilibrium may exist or yield very low profits. This explanation is consistent with the fact that in most countries with zero or low termination rates, the RPP retail regime is predominant (see Ofcom, 2009, Annex 8.1).

Next we discuss equilibrium profit and welfare by means of two numerical examples. We assume $n = 4$, $c_O = c_T = 0.5$, and $q(p) = p^{-2}$. In case (a), $\beta = 0.8$; in case (b) $\beta = 0.4$. Fig. 2 shows the equilibrium profit as a function of termination mark-up m for the CPP and RPP equilibria. In case (a) the CPP equilibrium exists for any $m \geq -0.5$, while the RPP equilibrium does not exist for $m > -0.42$. In case (b) equilibrium existence requires $m \geq -0.32$ under CPP and $m \leq -0.17$ under RPP.

One observes that profit in the CPP equilibrium decreases as termination mark-up is lowered below cost. At some point firms may prefer to switch to the RPP equilibrium. As the termination rate is lowered all the way to zero (Bill and Keep), profit in the RPP equilibrium increases and can even be higher than the maximum profit in any CPP equilibrium (see Fig. 2(b)).

In order to see how efficiency is affected, we plot off-net call volume in Fig. 3. The top level indicates the efficient call volume, which cannot be achieved by any termination mark-up for either type of equilibrium. The lower the termination mark-up, the more efficient is the CPP equilibrium and the less efficient is the RPP equilibrium. However,



(a) $\beta = 0.8$ (b) $\beta = 0.4$

Figure 3: Call volume in CPP and RPP equilibrium ($n = 4$, $c_O = c_T = 0.5$, $q(p) = p^{-2}$)

depending on the level of the termination mark-up either CPP or RPP may be better. In case (a), CPP is always outperformed by RPP in terms of total welfare, but in case (b), there are termination mark-ups where CPP outperforms RPP.

When firms can freely negotiate a reciprocal termination rate, they would agree on one above cost in case (a) (and play CPP) and on Bill and Keep in case (b) (and play RPP). In both cases efficiency would be very low and a regulated termination rate at cost would improve welfare.

A particularly interesting comparison is the one between the (approximate) US scenario of Bill and Keep ($m = -c_T$) and RPP and the (future or current) situation in the EU with termination regulated at cost ($m = 0$) and CPP. Figs. 2 and 3 show that the EU scenario is more efficient but less profitable in case (b), while it is less efficient but more profitable in case (a). We now compare both scenarios in terms of producer, consumer and total surplus for more general constant elasticity demand functions and call externalities, fixing only $c_O = c_T = c/2$ and the number of firms $n \geq 3$.

In the US scenario, off-net call and reception prices are then $\hat{p}^{\text{US}} = \hat{r}^{\text{US}} = \beta(n-1)c/(2(n\beta - 1))$, so that off-net call volume is $\hat{q}^{\text{US}} = q(\hat{r}^{\text{US}}/\beta)$. Under the EU scenario off-net call price is $\hat{p}^{\text{EU}} = (n-1)c/(n-1-\beta)$ and off-net call volume equals $\hat{q}^{\text{EU}} = q(\hat{p}^{\text{EU}})$. Call volume is thus higher (and more efficient) in the European scenario if and only if $\beta < (n+1)/(2n+1) \equiv \beta^W$.

Using $q(p) = p^{-\eta}$ and the profit expressions from Propositions 1 and 2, one verifies that when the two scenarios are equally efficient (i.e., $\beta = \beta^W$), European producers earn higher profits than their US counterparts, while European consumers obtain less surplus. It follows that there are cutoffs $\beta^P < \beta^{CS} < \beta^W$ such that industry profit (consumer surplus) is higher in Europe when $\beta > \beta^P$ ($\beta < \beta^{CS}$).

Proposition 3. *Let $n \geq 3$ and $q(p) = p^{-\eta}$ with $\eta > 1$. Then there exist cut-off values $1/(n-1) < \beta^P < \beta^{CS} < \beta^W = (n+1)/(2n+1)$ such that*

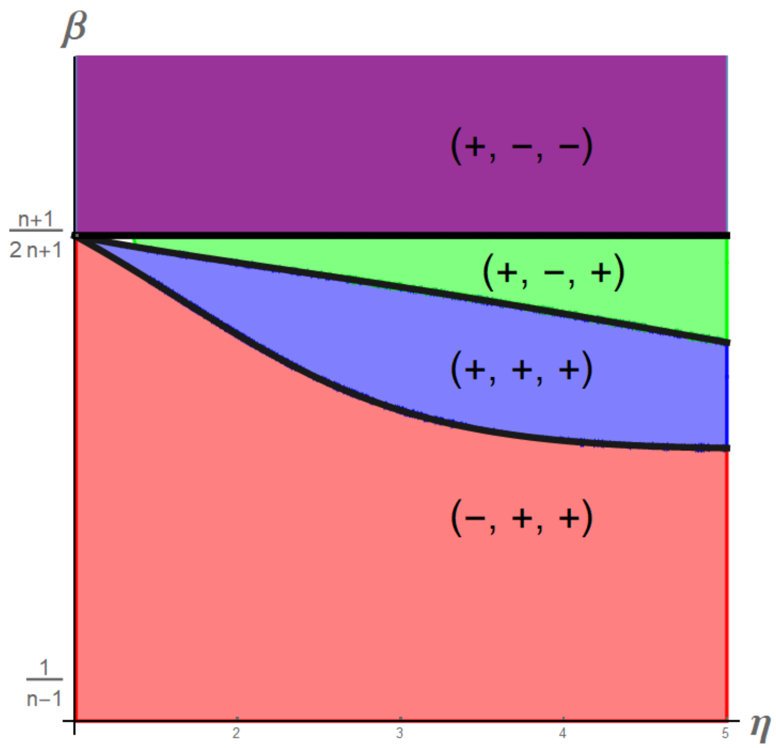


Figure 4: Producer, consumer and total surplus comparison between European and US scenario. (A '+' sign indicates that the relevant welfare measure is higher under the European scenario.)

[i] Producer surplus is higher in the EU than in the US scenario when $\beta > \beta^P$.

[ii] Consumer surplus is higher in the EU than in the US scenario when $\beta < \beta^{CS}$.

[iii] Total surplus is higher in the EU than in the US scenario when $\beta < \beta^W$.

Fig. 4 illustrates Proposition 3 by identifying different regions in the (η, β)-space: the sign '$+$' indicates when, respectively, producer, consumer or total surplus are higher under the European scenario.

5 Elastic subscription demand

Our analysis thus far assumes inelastic subscription demand. This is specially useful to understand the current state of affairs where penetration rates are very high, both in Europe and the US. However, in order to understand better the historical process of mobile take up, and to draw lessons for countries were penetration is still far from saturated, we here discuss the consequences of CPP versus RPP for penetration rates and equilibrium prices. Moving away from inelastic subscription demand, by allowing for an outside option worth w_0, makes the analysis much harder. On-net prices are still efficient, but it is not possible to obtain explicit expressions for equilibrium off-net prices, fixed fees or penetration rates. The reason for this is that one can determine the on-net price by keeping total penetration and relative market shares constant by means of an adjustment of the fixed fee. However, for off-net prices this trick does not work because the prices affect the surplus of rivals' customers and therefore total penetration will change when the adjustment of the fixed fee is used to keep relative market shares constant. Hence, off-net prices and fixed fees are only implicitly defined by first-order conditions. Nevertheless, these suffice to derive comparative statics and numerical expressions in particular examples (See also Hurkens and Jeon, 2012). An advantage of the elastic subscription demand model is that firms have less incentives to cause connectivity breakdown because that may lower the absolute number of own subscribers despite the possibility of increasing relative market share. Equilibrium existence is thus less problematic than under inelastic subscription demand.

Next, we derive the properties of CPP and RPP equilibra and then illustrate the effects of termination rates under CPP and RPP regimes on industry profit, penetration and prices using some numerical examples.

CPP

Now we characterize the CPP equilibrium when subscription demand is elastic. To reduce notation we define indirect utility in CPP by $v(p) = (1+\beta)u(q(p)) - pq(p)$.

Proposition 4. *In a symmetric CPP equilibrium under elastic subscription demand, on-net call price is $p^* = c/(1+\beta)$ while the number of subscribers per network, α^*, off-net call price, $\hat{p}^*$, and fixed fee F^* are implicitly given by the following system of equations*

$$0 = \hat{p}^* - c - m + \frac{\alpha^* \beta \hat{p}^*}{1-\alpha^*}\left[-1 + (1-n\alpha^*)\frac{R(\hat{p}^*)}{\mu}\right] \tag{12}$$

$$F^* = f + \frac{\mu}{1-\alpha^*} - 2\alpha^* R(p^*) - R(\hat{p}^*)(n-1)\frac{\alpha^*(1-2\alpha^*)}{1-\alpha^*} \tag{13}$$

$$F^* = \alpha^* v(p^*) + (n-1)\alpha^* v(\hat{p}^*) - w_0 - \mu \log\left(\frac{\alpha^*}{1-n\alpha^*}\right) \tag{14}$$

Equilibrium profit is equal to

$$\pi^* = \alpha^*\left(\frac{\mu}{1-\alpha^*} - \alpha^* R(p^*) + \frac{(n-1)\alpha^{*2}}{1-\alpha^*}R(\hat{p}^*)\right). \tag{15}$$

Note that, unsurprisingly, as $w_0 \to -\infty$, α^* tends to $1/n$ and expressions (12), (13) and (15) tend to the inelastic subscription demand equivalent expressions (6), (7) and (8). In comparison with the inelastic case, off-net price is now lower. We now examine how the termination rate affects price and penetration.

Proposition 5 (Comparative statics)**.** *For $|m|$ not too large and μ sufficiently large, an increase in m leads to higher $\hat{p}^*$ and lower α^*.*

This reconfirms the result of Hurkens and López (2014a, Prop. 5(i)) (obtained without call externality) that a network externality surcharge, as applied in the UK between 2003 and 2008, and in other European countries, in fact has a negative effect on penetration.

RPP

Now we characterize the RPP equilibrium when subscription demand is elastic. To reduce notation we define indirect utility in RPP by $\tilde{v}(r) = (1+\beta)u(q(r/\beta)) - 2rq(r/\beta)$.

Proposition 6. *In a symmetric RPP equilibrium under elastic subscription demand, on-net call prices are $p^* = r^* = \beta c/(1+\beta)$ while the number of subscribers per network, α^*, off-net prices, $\hat{p} = \hat{r} = \hat{r}^*$, and fixed fee $\tilde{F}^*$ are implicitly given by*

$$0 = \hat{r}^* + m + \frac{\alpha^*(1-\beta)\hat{r}^*}{(1-\alpha^*)\beta}\left[-1 + (1-n\alpha^*)\frac{\tilde{R}(\hat{r}^*)}{\mu}\right] \tag{16}$$

$$\tilde{F}^* = f + \frac{\mu}{1-\alpha^*} - 2\alpha^*\tilde{R}(r^*) - \tilde{R}(\hat{r}^*)(n-1)\frac{\alpha^*(1-2\alpha^*)}{1-\alpha^*} \tag{17}$$

$$\tilde{F}^* = \alpha^*\tilde{v}(r^*) + (n-1)\alpha^*\tilde{v}(\hat{r}^*) - w_0 - \mu\log\left(\frac{\alpha^*}{1-n\alpha^*}\right) \tag{18}$$

Equilibrium profit is equal to

$$\tilde{\pi}^* = \alpha^*\left(\frac{\mu}{1-\alpha^*} - \alpha^*\tilde{R}(r^*) + \frac{(n-1)\alpha^{*2}}{1-\alpha^*}\tilde{R}(\hat{r}^*)\right). \tag{19}$$

Note that for $m = 0$ the solution entails free off-net calls ($\hat{r}^* = 0$). For $m > 0$ there is no interior solution if negative prices are not allowed. Free off-net calls would be part of a (constrained) equilibrium then as well, but we will henceforth restrict attention to $m < 0$. Observe that for $\beta = 1$, $\hat{r}^* = -m$ and that for $\beta < 1$ any solution must have $\hat{r}^* > -m$: reception is charged above perceived marginal cost. The reason is that a raise in the reception charge hurts subscribers from rival networks as they can place less calls. However, under elastic subscription the incentive to raise price is not as strong as under inelastic subscription, where off-net reception price is given by Eq. (9). This does not mean, though, that under elastic subscription connectivity breakdown can never occur. For very low values of the call externality it still may be optimal to choke off-net traffic.

As $w_0 \to -\infty$, α^* tends to $1/n$ and expressions (16), (17) and (19) tend to the inelastic subscription demand equivalent expressions (9), (10) and (11). We now show that under RPP, an increase in termination rate above zero improves efficiency: it increases call volume and penetration.

Proposition 7 (Comparative statics)**.** *For $|m+c_T|$ not too large and μ sufficiently large, an increase in m leads to lower $\hat{r}^*$ and higher α^*.*

Hence, the apparent perception that RPP is better because it is associated with Bill and Keep termination regimes is misplaced. Not only do positive termination rates lower call prices and increase call volume, they also increase penetration.

Comparing EU and US

To provide some more insight in the differences between the European scenario, where firms play CPP equilibrium and termination rates were initially very high (about 20 cents per minute) but reduced to cost by regulation over time, and the US scenario, where firms play RPP and termination is determined by Bill and Keep, we calculate minutes of use (MoU), monthly bill (BILL), average revenue per minute (ARPM = BILL/MoU), penetration rate (PEN), industry profit (PROF) and total welfare (TW) for a numerical example. Table 1 reports results for a triopoly based on the following parameters, borrowed from the calibration performed in Hurkens and López (2012) for the Spanish market: $q(p) = a - bp$ with $a = 654.9$, $b = 2012.7$, $c = 2c_T = 0.049$, $f = 0$. We do this for three levels of call externality β. CPP_1 and CPP_2 refer to CPP equilibria with $m = 2c$ and $m = 0$, respectively, while RPP refers to RPP equilibria when $m = -c_T$ (Bill and Keep).

We have verified that both CPP and RPP equilibria without connectivity breakdown exist even when the call externality is low ($\beta = 1/3$). Under inelastic subscription demand they do not exist. This illustrates equilibrium existence is a less serious concern under elastic demand. It also suggests that call externalities are not extremely low, given that connectivity breakdown has not been observed in practice.

Notice that penetration under CPP_2 is about 10 per cent higher than under CPP_1. This explains why industry profit (and also total welfare) is higher under CPP_2. With inelastic subscription demand firms would actually have higher profits under CPP_1. One should recall that our model does not include fixed line operators or allow consumers to subscribe to several networks to avoid paying high off-net prices. In the early years, termination rates were high in Europe and believed to contribute to fast growth in mobile penetration, because they induced mobile operators to subsidize hand sets heavily.

β	1/3			2/3			1		
regime	CPP_1	CPP_2	RPP	CPP_1	CPP_2	RPP	CPP_1	CPP_2	RPP
MoU	615.94	906.37	581.94	623.99	975.36	1035.40	567.14	977.74	1159.90
BILL	89.43	93.58	76.91	94.58	101.18	86.59	95.12	106.85	87.66
ARPM	0.145	0.103	0.132	0.152	0.104	0.084	0.168	0.109	0.076
PEN	0.74	0.82	0.74	0.81	0.90	0.90	0.84	0.94	0.96
PROF	55.34	58.25	53.86	64.19	69.28	68.05	67.99	77.99	70.34
TW	123.56	142.93	120.68	147.10	182.61	185.31	158.70	219.32	228.49

Table 1: CPP_1 ($m = 2c$) vs CPP_2 ($m = 0$) vs RPP ($m = -c_T$) in triopoly, $\mu = 50$

Comparing CPP_2 with RPP, we observe that RPP performs slightly better in terms of penetration (and thus consumer welfare) and total welfare when $\beta \geq 2/3$. Industry profit is always higher under CPP_2.

6 Concluding remarks

We compared the CPP equilibrium (Europe) with the RPP equilibrium (US). Full efficiency is impossible under either retail regime. Under CPP, a reduction in termination rates induces lower call price, higher fixed fee and higher welfare. Firms switch to RPP before the termination rate reaches the efficient level. Under RPP, however, a reduction in termination rate leads to higher prices and profits, and to lower welfare. Efficiency now requires generally strictly positive termination rates. When termination rates are relatively high, firms would stop charging for the reception of calls (*i.e.*, switch to CPP).

Interestingly, both the EU and Ofcom, the UK regulator, have considered the possibility that termination rates below cost or equal to zero (Bill and Keep) might lead firms to switch to RPP regimes. We showed this is indeed possible but not warranted: the CPP equilibrium may exist even under Bill and Keep. On the other side of the ocean, back in 1999, US operators argued that CPP regimes would benefit low income consumers and urged the FCC to refrain from regulating termination rates (see Crandall and Sidak, 2004). The FCC eventually dismissed the idea but our analysis explains why operators could be interested in moving to a CPP regime with unregulated termination rates.

We showed that, depending on the strength of the call externality, the US scenario, consisting of a Bill and Keep termination regime combined with RPP retail tariffs, can be less or more efficient than the EU scenario with cost-based termination. In particular, we explained that letting firms freely negotiate reciprocal termination rates does generally not lead to (near) efficient outcomes: the interests of firms and regulators are not aligned, even when RPP regimes are used, contradicting results by Cambini and Valletti (2008) and Hoernig (2016). One exception is the extreme case where receivers value calls as much as callers ($\beta = 1$). Bill and Keep and RPP would then lead to efficiency,[19] but only if firms can be prevented to coordinate on the more lucrative CPP equilibrium. In order to give more conclusive policy recommendations, it would be extremely important to have a better idea of how strong call externalities actually are. This is presumably an empirical matter.[20]

Our results may be of limited importance for future policies in the EU and the US,

[19] Degraba (2003) obtains a similar result but does not consider competing networks.

[20] Hurkens and López (2012) calibrate welfare gains from regulation in the Spanish telecom market, while Harbord and Hoernig (2015) do so for the UK. In both cases a CPP regime is assumed and results depend strongly on the strength of the call externality. Rojas (2017) estimates a call externality β between 0.41 and 0.68 based on non-incentivized questionnaires in Ecuador. Using a similar methodology, Sobolewski and Czajkowski (2018) estimate a call externality between 0.31 and 0.53 in Poland.

because technological changes have led to estimated costs of termination close to zero, reducing the difference between them. Nevertheless, even at present one observes high termination rates in, for example, Switzerland (not being part of the EU) and Uruguay. We showed that, while there is no hope to achieve first-best efficiency by regulating termination rates, the European regulatory approach of reducing termination rates is welfare-enhancing and profit-reducing, explaining the continued opposition by large operators (see Crandall and Sidak, 2004). Average termination rates have been reduced from above 20 cents per minute in 2000 till below 1 cent per minute in 2020. The EU recommendation in 2009 (EC, 2009) to regulate rates at marginal cost, rather than at fully distributed costs, came perhaps a bit late and did not go far enough, taking into account the role of call externalities. We showed that the externality surcharge, as applied in the UK between 2003 and 2008, has a negative effect on penetration and welfare.

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Appendix A Proofs for section 3

Proof of Proposition 1. We already established that optimal on-net prices maximize welfare so that in a CPP equilibrium $r = 0$ and $p = p^* = c/(1+\beta)$. Since subscription demand is assumed inelastic and the off-net call price $\hat{p}_i$ affects all rivals in the same way in a symmetric equilibrium, one can calculate the optimal off-net call price of network i by keeping market shares constant (by adjusting F_i accordingly). Fixing $\hat{p}_j = \hat{p}^*$ and $F_j = F^*$ for all $j \neq i$, the profit of network i is equal to

$$\pi_i = \alpha_i \Big[\alpha_i (p^* - c) q(p^*) + (1-\alpha_i)(\hat{p}_i - c - m) q(\hat{p}_i) + (1-\alpha_i) m q(\hat{p}^*) + F_i - f \Big],$$

where F_i is such that the expected surplus from subscribing to network i is equal to the expected surplus from subscribing to any other network. In a symmetric equilibrium all consumers expect all networks to be of size $\alpha_i^e = 1/n$ so that

$$\begin{aligned} F_i = {} & \frac{n-1}{n} \Big[u(q(\hat{p}_i)) - \hat{p}_i q(\hat{p}_i) - (u(q(\hat{p}^*)) - \hat{p}^* q(\hat{p}^*)) \Big] \\ & + \frac{1}{n} \Big[\beta u(q(\hat{p}^*)) - \beta u(q(\hat{p}_i)) \Big] + F^*. \end{aligned}$$

Observe that

$$\frac{\partial F_i}{\partial \hat{p}_i} = \frac{n-1}{n} [-q(\hat{p}_i)] - \frac{1}{n} [\beta \hat{p}_i q'(\hat{p}_i)].$$

The first-order condition reads

$$\begin{aligned} 0 = \frac{\partial \pi_i}{\partial \hat{p}_i} &= \alpha_i \left[(1-\alpha_i)[q(\hat{p}_i) + (\hat{p}_i - c - m) q'(\hat{p}_i)] - \frac{n-1}{n} q(\hat{p}_i) - \frac{1}{n} \beta \hat{p}_i q'(\hat{p}_i) \right] \\ &= \alpha_i q'(\hat{p}_i) \left[(1-\alpha_i)(\hat{p}_i - c - m) - \frac{1}{n} \beta \hat{p}_i \right] + \alpha_i q(\hat{p}_i) \left(\frac{1}{n} - \alpha_i \right), \end{aligned} \tag{20}$$

so that in a symmetric equilibrium (where $\alpha_i = 1/n$) with $q'(\hat{p}_i) \neq 0$, we must have

$$(n - 1 - \beta)\hat{p}_i - (n-1)(c+m) = 0.$$

The second-order derivative of profits, evaluated at the solution of the first-order condition, reads

$$\frac{\partial^2 \pi_i}{\partial \hat{p}_i^2} = \frac{q'(\hat{p}_i)}{n^2} (n - 1 - \beta) < 0$$

for all $\beta < 1$ and $n \geq 2$. Hence, in a symmetric CPP equilibrium we have

$$\hat{p}^* = \frac{(n-1)(c+m)}{n-1-\beta}$$

Substituting this price into the profit function yields

$$\pi_i = \alpha_i[\alpha_i(p^* - c)q(p^*) + (1 - \alpha_i)(\hat{p}^* - c)q(\hat{p}^*) + F_i - f].$$

To find the equilibrium fixed fee we solve the first-order condition[21]

$$0 = \frac{\partial \pi_i}{\partial F_i} = -\frac{\alpha_i(1-\alpha_i)}{\mu}(2\alpha_i(p^* - c)q(p^*) + (1 - 2\alpha_i)(\hat{p}^* - c)q(\hat{p}^*) + F_i - f) + \alpha_i.$$

At a symmetric equilibrium $\alpha_i = 1/n$ so that equilibrium fixed fee satisfies

$$F^* = f + \frac{n\mu}{n-1} - \frac{2}{n}(p^* - c)q(p^*) - \frac{n-2}{n}(\hat{p}^* - c)q(\hat{p}^*), \tag{21}$$

and equilibrium profit equals

$$\pi^* = \frac{\mu}{n-1} - \frac{1}{n^2}(p^* - c)q(p^*) + \frac{1}{n^2}(\hat{p}^* - c)q(\hat{p}^*). \tag{22}$$

We have derived a unique symmetric CPP equilibrium candidate, but we still need to show that no firm wants to deviate and set a tariff with $\hat{r}_i > 0$. Next we show that a necessary condition is that $\beta\hat{p}^* \geq -m$.

Note that raising $\hat{r}_i$ up to $\beta\hat{p}^*$ does not affect off-net traffic and does therefore not strictly increase profits. (The extra revenues raised from reception charges, $\beta\hat{p}^* q(\hat{p}^*)$, must be compensated by an equal reduction in the fixed fee F_i to keep market share constant at $1/n$.)

Suppose now that network i considers to raise the reception charge for off-net calls above $\beta\hat{p}^*$. Such a deviation makes the receivers of this network determine the volume of calls received from subscribers from rival networks. Moreover, considering a deviation with $\hat{r}_i > \beta\hat{p}^*$ may change market share, and thus also the optimal call prices p_i and $\hat{p}_i$. So consider a unilateral deviation $(p_i, \hat{p}_i, \hat{r}_i, F_i)$ from the symmetric CPP equilibrium candidate that results in market share α_i. Then the profit equals

$$\pi_i = \alpha_i(\alpha_i(p_i - c)q(p_i) + (1 - \alpha_i)(\hat{p}_i - c - m)q(\hat{p}_i) + (1 - \alpha_i)(\hat{r}_i + m)q(\hat{r}_i/\beta) + F_i - f).$$

One can keep market share constant at α_i by adjusting F_i when varying p_i, $\hat{p}_i$ or $\hat{r}_i$.

[21] We use here that $\partial \alpha_i / \partial F_i = -\alpha_i(1-\alpha_i)/\mu$ from Eq. (2).

In particular, doing so implies

$$\begin{aligned}
\frac{\partial F_i}{\partial p_i} &= \frac{1}{n}(\beta p_i q'(p_i) - q(p_i)) \\
\frac{\partial F_i}{\partial \hat{p}_i} &= \frac{n-1}{n}[-q(\hat{p}_i)] - \frac{1}{n}[\beta \hat{p}_i q'(\hat{p}_i)] \\
\frac{\partial F_i}{\partial \hat{r}_i} &= \frac{n-1}{n}[-q(\hat{r}_i/\beta)] - \frac{1}{n\beta^2}[(\hat{r}_i - \beta\hat{p}^*)q'(\hat{r}_i/\beta)]
\end{aligned}$$

Note that actual market share does not enter the above expressions because of the assumption of passive expectations. Now

$$\begin{aligned}
\frac{\partial \pi_i}{\partial p_i} &= \alpha_i q'(p_i)\left[(\alpha_i(p_i - c) + \frac{\beta}{n}p_i)\right] + \alpha_i q(p_i)(\alpha_i - \frac{1}{n}) \\
\frac{\partial \pi_i}{\partial \hat{p}_i} &= \alpha_i\left[(1-\alpha_i)[q(\hat{p}_i) + (\hat{p}_i - c - m)q'(\hat{p}_i)] - \frac{n-1}{n}q(\hat{p}_i) - \frac{1}{n}\beta\hat{p}_i q'(\hat{p}_i)\right] \\
&= \alpha_i q'(\hat{p}_i)\left[(1-\alpha_i)(\hat{p}_i - c - m) - \frac{1}{n}\beta\hat{p}_i\right] + \alpha_i q(\hat{p}_i)\left(\frac{1}{n} - \alpha_i\right) \\
\frac{\partial \pi_i}{\partial \hat{r}_i} &= \alpha_i\left[(1-\alpha_i)[q(\hat{r}_i/\beta) + (\hat{r}_i + m)q'(\hat{r}_i/\beta)/\beta] - \frac{n-1}{n}q(\hat{r}_i/\beta) - \frac{q'(\hat{r}_i/\beta)}{n\beta^2}(\hat{r}_i - \beta\hat{p}^*)\right] \\
&= \alpha_i\frac{q'(\hat{r}_i/\beta)}{\beta^2 n}\left[n\beta(1-\alpha_i)(\hat{r}_i + m) - \hat{r}_i + \beta\hat{p}^*\right] + \alpha_i q(\hat{r}_i/\beta)(\frac{1}{n} - \alpha_i)
\end{aligned}$$

We look to maximize π_i subject to $\hat{r}_i \geq \beta\hat{p}^*$. When the maximum is obtained at $\hat{r}_i = \beta\hat{p}^*$ with $\alpha_i = 1/n$, the CPP equilibrium candidate is indeed an equilibrium. Otherwise there is a strictly profitable deviation and there is no symmetric CPP equilibrium.

Note that at $\alpha_i = 1/n$, $\frac{\partial \pi_i}{\partial \hat{r}_i} > 0$ at $\hat{r}_i = \beta\hat{p}^*$ when $\beta\hat{p}^* + m < 0$. In this case even a marginal deviation above $\beta\hat{p}^*$ is profitable and no CPP equilibrium exists. So a necessary condition for the existence of a symmetric CPP equilibrium is $\beta\hat{p}^* + m \geq 0$, which is equivalent to $m \geq \underline{m}^{\text{CPP}}$, which we will henceforth assume.

A further necessary condition for the existence of a symmetric CPP equilibrium is that no firm wants to deviate to $\hat{r}_i = \infty$ keeping market share constant at $1/n$. Note that exactly when $(n-1)\beta < 1$ such a deviation hurts subscribers of network i less than those of rival networks, so that network i can increase the fixed fee by $\Delta F = (1 - (n-1)\beta)[u(q(\hat{p}^*)) - \hat{p}^* q(\hat{p}^*)]/n$ and keep market share constant. However, breaking down incoming traffic reduces profit of network i because $\beta\hat{p}^* + m \geq 0$. Hence, when $(n-1)\beta < 1$, existence of a CPP equilibrium requires

$$\Delta F \leq (\beta\hat{p}^* + m)q(\hat{p}^*)(n-1)/n$$

However, this condition is not always sufficient: one needs to calculate whether the optimal deviation to connectivity breakdown (with market share different from $1/n$) yields higher profit. ■

Proof of Proposition 2. We already established that optimal on-net prices maximize welfare so that in an RPP equilibrium $p = r = r^* = \beta c/(1+\beta)$. Since subscription demand is assumed inelastic and the off-net reception price $\hat{r}_i$ affects all rivals in the same way (in a symmetric equilibrium), one can calculate the optimal off-net reception price of network i by keeping market shares constant by adjusting F_i accordingly. Let us fix $\hat{p}_j = \hat{r}^*$ for all $j \in N$, and $\hat{r}_j = \hat{r}^*$ and $F_j = \tilde{F}^*$ for all $j \neq i$, the profit of network i is equal to

$$\pi_i = \alpha_i\Big[\alpha_i(2r^* - c)q(r^*/\beta) + (1-\alpha_i)(\hat{r}^* - c - m)q(\hat{r}^*/\beta) + (1-\alpha_i)(\hat{r}_i + m)q(\hat{r}_i/\beta) + F_i - f\Big],$$

where F_i is such that expected surplus from subscribing to network i is equal to the expected surplus obtained from subscribing to any other network:

$$\begin{aligned} F_i = &\frac{n-1}{n}\Big[(\beta u(q(\hat{r}_i/\beta)) - \hat{r}_i q(\hat{r}_i/\beta)) - (\beta u(q(\hat{r}^*/\beta)) - \hat{r}^* q(\hat{r}^*/\beta))\Big] \\ &+ \frac{1}{n}\Big[(u(q(\hat{r}^*/\beta)) - \hat{r}^* q(\hat{r}^*/\beta)) - (u(q(\hat{r}_i/\beta)) - \hat{r}^* q(\hat{r}_i/\beta))\Big] + \tilde{F}^*. \end{aligned}$$

Observe that

$$\frac{\partial F_i}{\partial \hat{r}_i} = \frac{n-1}{n}[-q(\hat{r}_i/\beta)] - \frac{1}{n\beta^2}[(\hat{r}_i - \beta\hat{r}^*)q'(\hat{r}_i/\beta)].$$

At a symmetric equilibrium (with market share α_i kept constant at $1/n$) the derivative of profits w.r.t. $\hat{r}_i$ is

$$\begin{aligned} \frac{\partial \pi_i}{\partial \hat{r}_i} &= \frac{1}{n}\left[\frac{n-1}{n}[q(\hat{r}_i/\beta) + (\hat{r}_i + m)q'(\hat{r}_i/\beta)/\beta - q(\hat{r}_i/\beta)] - \frac{1}{n}(\hat{r}_i/\beta - \hat{r}^*)q'(\hat{r}_i/\beta)/\beta\right] \\ &= \frac{q'(\hat{r}_i/\beta)}{\beta^2 n^2}\left[(n-1)\beta(\hat{r}_i + m) - \hat{r}_i + \beta\hat{r}^*\right]. \end{aligned}$$

Hence, if $(n-1)\beta - 1 < 0$ no interior solution exists (as the optimal reception charge is then either $\beta\hat{r}^*$ or ∞). In particular, for $n = 2$ and $\beta < 1$ no RPP equilibrium exists.

If $(n-1)\beta - 1 = 0$ a solution exists only if $\beta\hat{r}^* = -m$ (which requires $m \leq 0$).

If $(n-1)\beta - 1 > 0$ a unique interior solution is given by

$$(\beta(n-1) - 1)\hat{r}_i + \beta(n-1)m + \beta\hat{r}^* = 0.$$

so that indeed in a symmetric RPP equilibrium $\hat{r}^*$ is given by (9).

Substituting the prices into the profit function and maximizing with respect to the fixed fee yields

$$\tilde{F}^* = f + \frac{n\mu}{n-1} - \frac{n-2}{n}(2\hat{r}^* - c)q(\hat{r}^*/\beta), \tag{23}$$

so that equilibrium profit equals

$$\tilde{\pi}^* = \frac{\mu}{n-1} + \frac{1}{n^2}(2\hat{r}^* - c)q(\hat{r}^*/\beta). \tag{24}$$

We now check whether a firm i may have an incentive to raise the off-net call price $\hat{p}_i$ above $\hat{r}^*/\beta$. Such a deviation makes the callers of this network determine the volume of off-net calls. Profit of firm i is then equal to

$$\pi_i = \alpha_i(\alpha_i(2r_i - c)q(r_i/\beta) + (1-\alpha_i)(\hat{p}_i - c - m)q(\hat{p}_i) + (1-\alpha_i)(\hat{r}^* + m)q(\hat{r}^*/\beta) + F_i - f).$$

As before, one can keep market share constant by adjusting F_i accordingly. That is

$$\begin{aligned} F_i = & \frac{n-1}{n}\left[(u(q(\hat{p}_i)) - \hat{p}_i q(\hat{p}_i)) - (u(q(\hat{r}^*/\beta)) - \hat{p}^* q(\hat{r}^*/\beta))\right] \\ & + \frac{1}{n}\left[(\beta u(q(\hat{r}^*/\beta)) - \hat{r}^* q(\hat{r}^*/\beta)) - (\beta u(q(\hat{p}_i)) - \hat{r}^* q(\hat{p}_i))\right] + \tilde{F}^* \end{aligned}$$

Observe that

$$\frac{\partial F_i}{\partial \hat{p}_i} = \frac{n-1}{n}[-q(\hat{p}_i)] - \frac{1}{n}[(\beta\hat{p}_i - \hat{r}^*)q'(\hat{p}_i)].$$

$$\begin{aligned} \partial\pi_i/\partial\hat{p}_i &= \alpha_i\left[\frac{n-1}{n}[q(\hat{p}_i) + (\hat{p}_i - c - m)q'(\hat{p}_i) - q(\hat{p}_i)] - \frac{1}{n}(\beta\hat{p}_i - \hat{r}^*)q'(\hat{p}_i)\right] \\ &= \alpha_i\frac{q'(\hat{p}_i)}{n}\left[(n-1-\beta)\hat{p}_i - (n-1)(c+m) + \hat{r}^*\right] \end{aligned}$$

so that the first-order condition holds when

$$(n-1-\beta)\hat{p}_i - (n-1)(c+m) + \hat{r}^* = 0.$$

Note that the second-order derivative of profits, evaluated at the solution of the first-order condition, reads

$$\frac{\partial^2\pi}{\partial\hat{p}_i^2} = \alpha_i\frac{q'(\hat{p}_i)}{n}(n-1-\beta) < 0$$

for all $\beta < 1$ and $n \geq 2$. A profitable (marginal) deviation above $\hat{r}^*/\beta$ thus exists whenever $\partial\pi/\partial\hat{p}_i > 0$, when evaluated at $\hat{p}_i = \hat{r}^*/\beta$. So a necessary condition for the candidate equilibrium to be an equilibrium is that $\hat{r}^* \geq \beta(c+m)$. This is equivalent to $m \leq \overline{m}^{RPP}$. ■

Appendix B Proofs for section 5

With elastic subscription demand consumers can choose to opt out and receive some utility w_0. Let $N = \{1, ..., n\}$. The expression for subscription surplus w_i from (1) becomes

$$\begin{aligned} w_i &= \alpha_i^e(U(p_i,r_i)+\widetilde{U}(p_i,r_i)-(p_i+r_i)D(p_i,r_i))-F_i \\ &\quad+\sum_{j\in N\setminus\{i\}}\alpha_j^e(U(\hat{p}_i,\hat{r}_j)-\hat{p}_iD(\hat{p}_i,\hat{r}_j)) \\ &\quad+\sum_{j\in N\setminus\{i\}}\alpha_j^e(\widetilde{U}(\hat{p}_j,\hat{r}_i)-\hat{r}_iD(\hat{p}_j,\hat{r}_i)) \end{aligned} \tag{25}$$

Subscription to network i attracts now α_i consumers where

$$\alpha_i=\frac{\exp[w_i/\mu]}{\sum_{k=0}^n\exp[w_k/\mu]} \tag{26}$$

It is easily verified that for all $i,j\in N$, $t\in\{F,p,r,\hat{p},\hat{r}\}$

$$\frac{\partial\alpha_i}{\partial t_j}=\frac{\alpha_i(1-\alpha_i)}{\mu}\frac{\partial w_i}{\partial t_j}-\frac{\alpha_i}{\mu}\sum_{k\in N\setminus\{i\}}\alpha_k\frac{\partial w_k}{\partial t_j}. \tag{27}$$

Profit expression (4) becomes

$$\begin{aligned} \pi_i &= \alpha_i\Big[\alpha_i(p_i+r_i-c)D(p_i,r_i)+F_i-f\Big]+ \\ &\quad \alpha_i\Big[(1-\alpha_i-\alpha_0)(\hat{p}_i-c-m)D(\hat{p}_i,\hat{r}^*)+(1-\alpha_i-\alpha_0)(\hat{r}_i+m)D(\hat{p}^*,\hat{r}_i)\Big] \end{aligned} \tag{28}$$

A change in the fixed fee or the prices for on-net traffic of network i does not affect the expected net surplus from subscribing to network $j\neq i$ (i.e. $\partial w_j/\partial t_i=0$ for $t\in\{F,p,r\}$). On the other hand, off-net prices of network i may affect the net utility from subscribing to a different network, through the effect on the volume of off-net calls. Let γ^* denote the expected size of each network in a symmetric equilibrium.

Note that

$$\frac{\partial w_i}{\partial p_i}=\gamma^*(\beta p_iq'(p_i)-q(p_i)) \tag{29}$$

$$\frac{\partial w_i}{\partial \hat{p}_i}=-(n-1)\gamma^*q(\hat{p}_i) \tag{30}$$

$$\frac{\partial w_i}{\partial F_i}=-1 \tag{31}$$

$$\frac{\partial w_j}{\partial p_i}=0 \tag{32}$$

$$\frac{\partial w_j}{\partial \hat{p}_i}=\gamma^*\beta\hat{p}_iq'(\hat{p}_i) \tag{33}$$

$$\frac{\partial w_j}{\partial F_i}=0. \tag{34}$$

Proof of Proposition 4

The first-order condition with respect to F_i yields

$$\begin{aligned}
0 = \frac{\partial \pi_i}{\partial F_i} =& \frac{\partial \alpha_i}{\partial F_i}\left(\frac{\pi_i}{\alpha_i}\right) + \alpha_i \left(\frac{\partial \alpha_i}{\partial F_i} R(p^*) + \sum_{j \in N \setminus \{i\}} \frac{\partial \alpha_j}{\partial F_i} R(\hat{p}_i) + 1\right) \\
=& -\frac{\alpha_i(1-\alpha_i)}{\mu}(2\alpha_i R(p^*) + (1-\alpha_i-\alpha_0)R(\hat{p}_i) + F_i - f) \\
&+ \alpha_i \left(\frac{\alpha_i(1-\alpha_i-\alpha_0)}{\mu} R(\hat{p}_i) + 1\right).
\end{aligned}$$

It follows then that in a symmetric equilibrium fixed fee must satisfy (13).

The first-order condition with respect to the off-net call price $\hat{p}_i$ yields

$$\begin{aligned}
0 = \frac{\partial \pi_i}{\partial \hat{p}_i} =& \frac{\partial \alpha_i}{\partial \hat{p}_i}\left(\frac{\pi_i}{\alpha_i}\right) + \alpha_i \left(\frac{\partial \alpha_i}{\partial \hat{p}_i} R(p^*) + \sum_{j \in N \setminus \{i\}} \frac{\partial \alpha_j}{\partial \hat{p}_i} R(\hat{p}_i)\right) \\
&+ \alpha_i \sum_{j \in N \setminus \{i\}} \alpha_j [q(\hat{p}_i) + (\hat{p}_i - c - m) q'(\hat{p}_i)] \\
=& \frac{\partial \alpha_i}{\partial \hat{p}_i}\left[2\alpha_i R(p^*) + \sum_{j \in N \setminus \{i\}} \alpha_j R(\hat{p}^*) + F_i - f\right] \\
&+ \alpha_i \sum_{j \in N \setminus \{i\}} \frac{\partial \alpha_j}{\partial \hat{p}_i} R(\hat{p}^*) \\
&+ \alpha_i \sum_{j \in N \setminus \{i\}} \alpha_j [q(\hat{p}_i) + (\hat{p}_i - c - m) q'(\hat{p}_i)]
\end{aligned}$$

Hence,

$$\begin{aligned}
0 =& \left(\frac{\alpha^*(1-\alpha^*)}{\mu}\frac{\partial w_i}{\partial \hat{p}_i} - \frac{(n-1)\alpha^{*2}}{\mu}\frac{\partial w_j}{\partial \hat{p}_i}\right)\left(\frac{\mu}{1-\alpha^*} + R(\hat{p}^*)\frac{(n-1)\alpha^{*2}}{1-\alpha^*}\right) \\
&+ (n-1)\alpha^* R(\hat{p}^*)\left(-\frac{\alpha^{*2}}{\mu}\frac{\partial w_i}{\partial \hat{p}_i} + \frac{\alpha^*(1-(n-1)\alpha^*)}{\mu}\frac{\partial w_j}{\partial \hat{p}_i}\right) \\
&+ (n-1)\alpha^{*2}[q(\hat{p}^*) + (\hat{p}^* - c - m) q'(\hat{p}^*)]
\end{aligned}$$

Straightforward algebra shows that this can be rewritten as

$$\begin{aligned}
0 =& \frac{\partial w_i}{\partial \hat{p}_i}\alpha^* + (n-1)\alpha^{*2}[q(\hat{p}^*) + (\hat{p}^* - c - m) q'(\hat{p}^*)] \\
&+ \frac{(n-1)\alpha^{*2}}{\mu}\frac{\partial w_j}{\partial \hat{p}_i}\left(\frac{-\mu}{1-\alpha^*} + R(\hat{p}^*)\frac{1-n\alpha^*}{1-\alpha^*}\right)
\end{aligned}$$

Substituting equations (30) and (33) this can then be rewritten as eq. (12).

Finally, from (26) we know that the number of subscribers per firm (denoted by α^*), must satisfy

$$\alpha^* = \frac{\exp[w^*/\mu]}{n\exp[w^*/\mu] + \exp[w_0/\mu]}.$$

Denoting indirect CPP utility by $v(p) = (1+\beta)u(q(p)) - pq(p)$, the above equation can be rewritten as

$$F^* = \alpha^* v(p^*) + (n-1)\alpha^* v(\hat{p}^*) - w_0 - \mu \log\left(\frac{\alpha^*}{1-n\alpha^*}\right) \tag{35}$$

which is (14).

■

Proof of Proposition 5

Let us define

$$\Phi(\alpha^*, \hat{p}^*) = \hat{p}^* - c - m + \frac{\alpha^*\beta\hat{p}^*}{1-\alpha^*}\left[-1 + (1-n\alpha^*)\frac{R(\hat{p}^*)}{\mu}\right] \tag{36}$$

$$\Psi(\alpha^*, \hat{p}^*) = f + \frac{\mu}{1-\alpha^*} - 2\alpha^* R(p^*) - R(\hat{p}^*)(n-1)\frac{\alpha^*(1-2\alpha^*)}{1-\alpha^*} \tag{37}$$

$$- \left(\alpha^* v(p^*) + (n-1)\alpha^* v(\hat{p}^*) - w_0 - \mu\log\left(\frac{\alpha^*}{1-n\alpha^*}\right)\right) \tag{38}$$

CPP equilibrium is then implicitly defined $\Phi(\alpha^*, \hat{p}^*) = \Psi(\alpha^*, \hat{p}^*) = 0$.

It is straightforward to verify that (for small $|m|$, sufficiently large μ, $\hat{p}^* < p^M$ and $\alpha^* > 1/(n+1)$) partial derivatives satisfy

$$\Psi_{\alpha^*} > 0, \Psi_{\hat{p}^*} > 0, \Psi_m = 0$$

and

$$\Phi_{\alpha^*} < 0, \Phi_{\hat{p}^*} > 0, \Phi_m = -1$$

To be precise, we have

$$\Psi_{\alpha^*} = \frac{\mu}{(1-\alpha^*)^2} - 2R(p^*) - v(p^*) - (n-1)\frac{1-4\alpha^*+2\alpha^{*2}}{(1-\alpha^*)^2}R(\hat{p}^*) - (n-1)v(\hat{p}^*) + \frac{\mu}{\alpha^*(1-n\alpha^*)} > 0$$

$$\Psi_{\hat{p}^*} = -(n-1)\alpha^*\left[q'(\hat{p}^*)[(1+\beta)\hat{p}^* - c]\frac{1-2\alpha^*}{1-\alpha^*} - \frac{\alpha^* q(\hat{p}^*)}{1-\alpha^*}\right] > 0$$

$$\Phi_{\alpha^*} = \frac{\beta\hat{p}^*}{(1-\alpha^*)^2}\left[-1 + (1-2n\alpha^* + n\alpha^{*2})\frac{R(\hat{p}^*)}{\mu}\right] < 0$$

$$\Phi_{\hat{p}^*} = \frac{c+m}{\hat{p}^*} + \frac{\alpha^*\beta\hat{p}^*}{1-\alpha^*}(1-n\alpha^*)\frac{R'(\hat{p}^*)}{\mu} > 0$$

Hence, in $(\hat{p}^*, \alpha^*)$-space the curve $\Phi(\alpha^*, \hat{p}^*) = 0$ is upward sloping and the curve $\Psi(\alpha^*, \hat{p}^*) = 0$ is downward sloping. The results follow then straightforwardly from analyzing how each curve shifts when m is changed: An increase in m shifts the Φ curve down while holding the Ψ curve fixed, leading to an increase in $\hat{p}^*$ and a decrease in α^*. ∎

Proof of Proposition 6

In this case call volume is determined by the reception charge, i.e. $q(r/\beta)$. Note that in this case, at the symmetric equilibrium, we have

$$\frac{\partial w_i}{\partial r_i} = \gamma^* \left(-2q(r^*/\beta) + r^* q'(r^*/\beta) \frac{1-\beta}{\beta^2} \right) \tag{39}$$

$$\frac{\partial w_i}{\partial \hat{r}_i} = -2(n-1)\gamma^* q(\hat{r}^*/\beta) \tag{40}$$

$$\frac{\partial w_i}{\partial \tilde{F}_i} = -1 \tag{41}$$

$$\frac{\partial w_j}{\partial r_i} = 0 \tag{42}$$

$$\frac{\partial w_j}{\partial \hat{r}_i} = \gamma^* \hat{r}^* q'(\hat{r}^*/\beta) \frac{1-\beta}{\beta^2} \tag{43}$$

$$\frac{\partial w_j}{\partial \tilde{F}_i} = 0. \tag{44}$$

Let us now consider the first-order condition (evaluated at the symmetric equilibrium) with respect to the fixed fee F_i.

$$\begin{aligned} 0 = \frac{\partial \pi_i}{\partial F_i} =& \frac{\partial \alpha_i}{\partial F_i} \left(\frac{\pi_i}{\alpha_i} \right) + \alpha^* \left(\frac{\partial \alpha_i}{\partial F_i} \tilde{R}(r^*) + \sum_{j \in N \setminus \{i\}} \frac{\partial \alpha_j}{\partial F_i} \tilde{R}(\hat{r}^*) + 1 \right) \\ =& -\frac{\alpha^*(1-\alpha^*)}{\mu} (2\alpha^* \tilde{R}(r^*) + (n-1)\alpha^* \tilde{R}(\hat{r}^*) + F^* - f) \\ &+ \alpha^* \left(\frac{(n-1)\alpha^{*2}}{\mu} \tilde{R}(\hat{r}^*) + 1 \right). \end{aligned}$$

In a symmetric equilibrium we must thus have

$$\tilde{F}^* = f + \frac{\mu}{1-\alpha^*} - 2\alpha^* \tilde{R}(r^*) - \tilde{R}(\hat{r}^*)(n-1) \frac{\alpha^*(1-2\alpha^*)}{1-\alpha^*} \tag{45}$$

which is just (17).

Let us now consider the first-order condition with respect to the off-net price $\hat{r}_i$.

$$
\begin{aligned}
0 = \frac{\partial \pi_i}{\partial \hat{r}_i} =& \frac{\partial \alpha_i}{\partial \hat{r}_i}\left(\frac{\pi_i}{\alpha^*}\right) + \alpha^*\left(\frac{\partial \alpha_i}{\partial \hat{r}_i}\tilde{R}(r^*) + \sum_{j\in N\setminus\{i\}} \frac{\partial \alpha_j}{\partial \hat{r}_i}\tilde{R}(\hat{r}^*)\right) \\
&+ \sum_{j\in N\setminus\{i\}} \alpha_j[2q(\hat{r}^*/\beta) + (\hat{r}^* + m)q'(\hat{r}^*/\beta)/\beta] \\
=& \frac{\partial \alpha_i}{\partial \hat{r}_i}\left(2\alpha^*\tilde{R}(r^*) + (n-1)\alpha^*\tilde{R}(\hat{r}^*) + \tilde{F}^* - f\right) \\
&+ \alpha^*\left[(n-1)\left(\frac{\partial \alpha_i}{\partial \hat{r}_i}\right)\tilde{R}(r^*)\right] \\
&+ \alpha^{*2}(n-1)[2q(\hat{r}^*/\beta) + (\hat{r}^* + m)q'(\hat{r}^*/\beta)/\beta]
\end{aligned}
$$

Substituting equation (17), using equation (27) and considering being at a symmetric equilibrium we obtain

$$
\begin{aligned}
0 =& \left(\frac{\alpha^*(1-\alpha^*)}{\mu}\frac{\partial w_i}{\partial \hat{r}_i} - \frac{(n-1)\alpha^{*2}}{\mu}\frac{\partial w_j}{\partial \hat{r}_i}\right)\left(\frac{\mu}{1-\alpha^*} + \tilde{R}(\hat{r}^*)\frac{(n-1)\alpha^{*2}}{1-\alpha^*}\right) \\
&+ (n-1)\alpha^*\tilde{R}(\hat{r}^*)\left(-\frac{\alpha^{*2}}{\mu}\frac{\partial w_i}{\partial \hat{r}_i} + \frac{\alpha^*(1-(n-1)\alpha^*)}{\mu}\frac{\partial w_j}{\partial \hat{r}_i}\right) \\
&+ (n-1)\alpha^{*2}[2q(\hat{r}^*/\beta) + \frac{\hat{r}^* + m}{\beta}q'(\hat{r}^*/\beta)]
\end{aligned}
$$

Straightforward algebra shows that this can be rewritten as

$$
\begin{aligned}
0 =& \frac{\partial w_i}{\partial \hat{r}_i}\alpha^* + (n-1)\alpha^{*2}\left[2q(\hat{r}^*/\beta) + \frac{\hat{r}^* + m}{\beta}q'(\hat{r}^*/\beta)\right] \\
&+ \frac{(n-1)\alpha^{*2}}{\mu}\frac{\partial w_j}{\partial \hat{r}_i}\left(\frac{-\mu}{1-\alpha^*} + \tilde{R}(\hat{r}^*)\frac{1-n\alpha^*}{1-\alpha^*}\right)
\end{aligned}
$$

Substituting equations (40) and (43) this can then be rewritten as

$$
0 =(n-1)\alpha^{*2}q'(\hat{r}^*/\beta)\left(\frac{\hat{r}^* + m}{\beta} + \frac{\alpha^*\beta\hat{r}^*}{1-\alpha^*}\left[-1 + (1-n\alpha^*)\frac{\tilde{R}(\hat{r}^*)}{\mu}\right]\right) \qquad (46)
$$

so that (16) must hold.

From (2) we know that the number of subscribers per firm (denoted by α^*), must satisfy

$$
\alpha^* = \frac{\exp[w^*/\mu]}{n\exp[w^*/\mu] + \exp[w_0/\mu]}.
$$

Denoting indirect RPP utility by $\tilde{v}(r) = (1+\beta)u(q(r/\beta)) - 2rq(r/\beta)$, the above equation can be rewritten as

$$\tilde{F}^* = \alpha^* \tilde{v}(r^*) + (n-1)\alpha^* \tilde{v}(\hat{r}^*) - w_0 - \mu \log\left(\frac{\alpha^*}{1-n\alpha^*}\right) \tag{47}$$

which is just (18). ■

Proof of Proposition 7

Let us define

$$\begin{aligned}\widetilde{\Psi}(\alpha^*, \hat{r}^*) = & \left[f + \frac{\mu}{1-\alpha^*} - 2\alpha^* \tilde{R}(r^*) - \tilde{R}(\hat{r}^*)(n-1)\frac{\alpha^*(1-2\alpha^*)}{1-\alpha^*}\right] \\ & - \left[\alpha^* \tilde{v}(r^*) + (n-1)\alpha^* \tilde{v}(\hat{r}^*) - w_0 - \mu \log\left(\frac{\alpha^*}{1-n\alpha^*}\right)\right]\end{aligned} \tag{48}$$

$$\widetilde{\Phi}(\alpha^*, \hat{r}^*) = \frac{\hat{r}^* + m}{\beta} + \frac{\alpha^*(1-\beta)\hat{r}^*}{(1-\alpha^*)\beta^2}\left[-1 + (1-n\alpha^*)\frac{\tilde{R}(\hat{r}^*)}{\mu}\right] \tag{49}$$

The RPP equilibrium is implicitly defined by $\widetilde{\Psi}(\alpha^*, \hat{r}^*) = \widetilde{\Phi}(\alpha^*, \hat{r}^*) = 0$.

The comparative statics exercises are very similar to the case of CPP. We now have $\tilde{\Psi}_m = 0$ and $\tilde{\Phi}_m = 1/\beta > 0$ and

$$\begin{aligned}\tilde{\Psi}_{\alpha^*} = & \frac{\mu}{(1-\alpha^*)^2} - 2\tilde{R}(r^*) - \tilde{v}(r^*) - (n-1)\frac{1-4\alpha^*+2\alpha^{*2}}{(1-\alpha^*)^2}\tilde{R}(\hat{r}^*) - (n-1)\tilde{v}(\hat{r}^*) + \frac{\mu}{\alpha^*(1-n\alpha^*)} > 0 \\ \tilde{\Psi}_{\hat{r}^*} = & -(n-1)\alpha^*\left[\frac{q'(\hat{r}^*/\beta)}{\beta}\left[[2\hat{r}^* - c]\frac{1-2\alpha^*}{1-\alpha^*} + \frac{1-\beta}{\beta}\hat{r}^*\right] - \frac{\alpha^* q(\hat{r}^*/\beta)}{1-\alpha^*}\right] > 0 \\ \tilde{\Phi}_{\alpha^*} = & \frac{(1-\beta)\hat{r}^*}{(1-\alpha^*)^2\beta^2}\left[-1 + (1-2n\alpha^* + n\alpha^{*2})\frac{\tilde{R}(\hat{r}^*)}{\mu}\right] < 0 \\ \tilde{\Phi}_{\hat{r}^*} = & \frac{-m}{\beta\hat{r}^*} + \frac{\alpha^*(1-\beta)\hat{r}^*}{(1-\alpha^*)\beta^2}(1-n\alpha^*)\frac{\tilde{R}'(\hat{r}^*)}{\mu} > 0\end{aligned}$$

■