Who should pay for two-way interconnection?*

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Abstract

We analyze a symmetric oligopoly model in which mobile operators may charge subscribers for placing and receiving calls. A continuum of equilibria exists, some of which resemble the European CPP business model and some others resemble the US RPP business model. Efficiency can be achieved but not under these two business models. CPP with termination regulated at cost is more efficient than RPP with Bill and Keep arrangements when call externality is modest, and more profitable when either call externality is modest and call demand elasticity high or call externality is high and call demand elasticity low.

Key words: CPP, RPP, interconnection, call externality, regulation.

JEL Classification: D43, L13, L51.

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1 Introduction

The liberalization of telecommunication markets has undoubtedly led to increased consumer surplus through higher quality service, lower prices and more variety. At the same time the need for heavy-handed regulation of retail prices of incumbent monopolists was eliminated. However, it also brought about new regulatory issues, such as the one of interconnection. Whenever a customer of operator A wants to call a customer of competitor B, the latter has to provide a wholesale service called 'call termination'. Call termination is in fact a matter of two-way access because customers of B will also want to call customers of A. Who should pay for this two-way access? And, how much? Should the originating operator pay the full cost to the terminating operator, or should this cost be shared? How much of these costs will be passed on to consumers? Should the receiving party pay for termination? In this paper we address these questions by analyzing how termination costs and charges are passed on to subscribers when operators are free to set prices both for placing and receiving calls, as well as a (monthly) fixed fee. We also analyze the effects on private and social welfare.

We build upon Jeon et al. (2004) (hereafter JLT) but our model and analysis exhibit several important differences. First, JLT consider a duopoly while we allow for any number of firms. Second, JLT restrict attention to the equilibrium that is robust to adding vanishing noise to the utility from receiving a call. Third, JLT characterize equilibrium candidates by means of necessary local first-order conditions but only perform a partial analysis of sufficient conditions. This allows them to conclude that no termination rate can lead to an efficient outcome.[1] The impossibility to obtain efficiency is disturbing from a policy perspective.[2] It is conceivable that this problem is caused by JLT's restriction to equilibria robust to vanishing noise. We therefore analyze the full set of equilibria by presenting necessary and sufficient conditions. This also allows us to compare and understand the development of termination and retail prices in Europe and the US in the past and to draw policy conclusions for the future, still very relevant in many countries in Latin-America.

While JLT's selection criterion certainly is appealing[3], alternative equilibria may be plausible as well, or even more so.[4] In particular, initial conditions and path dependence may have played an important role in equilibrium coordination. In the early years of mobile telephony, most calls received were originated from the fixed network. In Europe,

[1]The reason is that for the termination rate that would make the unique equilibrium candidate efficient, the candidate fails to be actually an equilibrium because each firm has a profitable global deviation: provoking connectivity breakdown by raising the reception charge to infinity. Moreover, for other, inefficient, termination rates the equilibrium candidate has asymptotic connectivity breakdown as the call externality becomes very strong.

[2]Moreover, connectivity breakdowns are more of a theoretical curiosity than of practical relevance.

[3]Noisy receiver utility explains why sometimes the caller and sometimes the receiver ends a call.

[4]Equilibrium candidates robust to vanishing noise have (off-net) calling price strictly above reception charge, which is inconsistent with the observation that in RPP countries both charges are equal.

mobile operators obtained the bulk of their revenues from the unregulated (and thus high) termination rates paid by the fixed network and had therefore little incentive to charge own subscribers for receiving calls, as this would reduce incoming traffic and wholesale revenue. As mobile penetration increased and termination charges became regulated and decreased over time, these incentives have changed but it may have been difficult or even impossible to switch away from this voluntary Calling-Party-Pays business model when users are accustomed to not pay for reception. On the other hand, in the US, termination rates on mobile and fixed networks were forced to be reciprocal and were thus low from the initial years of mobile telephony. Mobile operators needed to recover their origination and termination costs from own subscribers and charged them the *same* price for placing and receiving calls. This particular Receiver-Party-Pays business model (which we call RPP*) has been applied since then.[5,6]

In contrast with JLT, we show that efficiency is possible. There exists at most one equilibrium that leads to efficient usage for off-net traffic. (Unsurprisingly, in *any* equilibrium on-net prices lead to fully efficient on-net traffic, independent of the termination charge.) Any other equilibrium leads to inefficiently low call volume.[7] In the efficient equilibrium off-net prices are such that caller and receiver share the total cost of a call in the same proportion they benefit from it.[8] This equilibrium can only arise for one particular value of the termination charge (strictly positive but below cost) which depends on this proportion β, usually referred to as the call externality. Moreover, existence of the efficient equilibrium requires that $(n-1)\beta > 1$, where n is the number of firms. Our generalization to an oligopoly is thus very important because the condition is not satisfied for $n = 2$ and $\beta \leq 1$. Fortunately, the necessary condition is likely satisfied in practice, because all OECD countries have at least three mobile operators (see OECD, 2012).

In terms of private welfare properties, it is important to remark that we assume that consumers form passive expectations about market shares (as in Hurkens and López, 2014) whereas JLT assume that consumers form responsive expectations.[9] The type of expectations is irrelevant for usage prices and thus for total welfare, but is crucial for determining fixed fees, and thus profits. Profits do not directly depend on termination charges because in equilibrium termination payments cancel out against termination rev-

[5]The term Receiver-Party-Pays generally refers to the case where the receiver pays some positive price that can be distinct from the price paid for placing a call.

[6]At present, both in Europe and the US many tariff plans offer a bucket of minutes for a flat fee but it is still true that only in the US minutes received count toward the budget.

[7]In contrast, when firms are not allowed to charge for reception, termination rates below cost may result in inefficiently high off-net call volume. (See Gans and King, 2001).

[8]This result is reminiscent of Degraba (2003). He shows that if firms set call and reception charges equal to perceived marginal cost, then there exists a termination charge that yields efficient call volume. For this termination charge (which is equal to zero for some parameter values) caller and receiver also share the cost of a call in proportion to their personal benefit. However, we show that firms will usually not set price equal to perceived marginal cost.

[9]Expectations about market shares are important when there is price discrimination between on- and off-net calls. Passive expectations do not react to price deviations while responsive expectations do.

enues. Profits only depend on the termination charge insofar as it affects total price for an off-net call (i.e., the sum of prices for placing and receiving a call) and the volume of off-net traffic. In particular, under passive expectations overall profit increases whenever profit from off-net traffic increases. This result is consistent both with the fact that operators in Europe, playing the voluntary CPP equilibrium, have opposed reductions in termination fees imposed by regulators, and that operators in the US, playing the RPP* equilibrium, reach bilateral agreements on Bill and Keep. Under responsive expectations, however, overall profit also depends on the difference between indirect utility from off- and on-net traffic. In particular, the more efficient is off-net traffic, the higher will be the equilibrium fixed fee and overall profit.[10] This effect is so strong that firms prefer the termination charge that induces efficient off-net traffic when consumers form responsive expectations. Results based on responsive beliefs are therefore inconsistent with observed behavior in Europe and the US.

Despite our efficiency possibility result (for an industry with at least three firms), there remain at least two serious difficulties for achieving first-best outcomes: the multiplicity of equilibria and the private incentives to agree on too low termination charges.

First, even if regulators can calculate and set the necessary termination charge, there is no guarantee that firms will play the efficient equilibrium. In particular, when firms coordinate on the voluntary CPP equilibrium, as firms do in Europe, efficiency is impossible. Similarly, when firms play the equilibrium with equal price for placing and receiving calls (that is, the RPP* equilibrium), as is usually done in the US, efficiency is impossible. However, if other equilibrium selection criteria are used, efficiency may obtain when the regulator sets the required termination charge. For example, if firms play the equilibrium that is robust to adding vanishing noise to the receiver's marginal utility (as in JLT), the efficient equilibrium is played. Also, if firms coordinate on the profit maximizing equilibrium for a given termination charge, efficiency results. The second concern is that when regulation consists in setting a price ceiling, firms may want to agree on termination charges strictly below the one that is compatible with efficiency. For example, Bill and Keep (i.e., no termination charge) may yield equilibrium prices that lead to higher industry profit but lower total welfare. This is true if firms play the RPP* equilibrium, the equilibrium robust to vanishing noise, or the joint profit maximizing equilibrium.

Hence, neither a regulator setting a ceiling for termination charges nor letting firms negotiate reciprocal termination charges will lead to efficiency. In particular, neither the European nor the US approach can lead to full efficiency. Nevertheless, it is an interesting exercise to see whether the European or the US approach is better in terms of generating consumer, producer or total surplus. We compare consumer, producer

[10] The intuition is that the higher is the indirect utility from off-net traffic, the more consumers prefer to join the smaller network (because more of its traffic will then be off-net). This softens competition for subscription.

and total surplus in the CPP equilibrium with termination priced at cost (which is the objective of the European Commission) with the RPP* equilibrium under Bill and Keep (which is the main scenario in the US). Consumer and total surplus are higher under the European scenario for relatively low (but not implausible) values of the call externality. Producer surplus is higher under the European scenario in two distinct regions: for low call externality and high elasticity and for high call externality and low elasticity. The intuition for these results is that for high (respectively, low) levels of the call externality the RPP* (respectively, CPP) regime leads to higher and more efficient call volumes sold at lower margins.

Related literature. This paper is certainly not the first to investigate the impact of termination rates on competition between telecommunication networks. We now explain how our analysis contributes and relates to the existing literature that starts with the seminal works of Armstrong (1998) and Laffont et al. (1998a,b)[11] where consumers derive no utility from receiving calls and firms always employ a CPP regime. Laffont et al. (1998b) consider the case when networks compete in nonlinear prices and can charge different prices for on- and off-net calls, as we do. They show that profit is strictly decreasing in termination charge. Building on their analysis, Gans and King (2001) show that firms strictly prefer termination charges below cost.[12] Firms would then set lower prices for off-net calls than for on-net calls. Berger (2005) considers the same 'mandatory' CPP setting but allows for call externalities. The termination charge that maximizes total welfare is then strictly below cost and, for sufficiently strong call externality, both firms and regulator prefer Bill and Keep over any positive termination charge.

These theoretical results are at odds with real world observations because regulators in CPP countries are typically concerned about too high termination charges and operators consistently oppose cutting termination rates.[13] A few recent papers reconcile real world observations with theory by changing the Laffont et al. (1998b) model in various ways (but still insist on zero prices for reception) and then show that firms do prefer termination rates above cost. In particular, Hurkens and López (2014) show this by assuming that consumers have passive expectations about network sizes. We show there that industry profit is increasing in termination charge and that efficiency is obtained when termination is priced below cost (when call externalities exist). The present paper also assumes

[11]For a complete review of the early literature on access charges see Armstrong (2002), Vogelsang (2003), and Peitz et al. (2004).

[12]The intuition for their results is that if termination charge is above cost, off-net calls will be more expensive than on-net calls. As there is a price differential between on- and off-net calls, consumers care about the size of each network (the so-called 'tariff-mediated network externalities'). In particular, they will be more eager to join the larger network. Consequently, acquisition costs are reduced, which in turn intensifies competition for subscribers and results in lower subscription fees and profits.

[13]Nevertheless, this result has been shown to be robust with respect to the number of networks (Calzada and Valletti, 2008), asymmetries (López and Rey, 2016), and elastic subscription demand Hurkens and Jeon (2012).

passive expectations but shows that if firms are allowed to charge for reception, the efficient 'mandatory' CPP equilibrium ceases to be a 'voluntary' CPP equilibrium because firms would necessarily charge a strictly positive price for reception if allowed to do so. Other papers that give explanations for why firms object against lowering termination charges are Armstrong and Wright (2009), Hoernig et al. (2014), Jullien et al. (2013), and Tangerås (2014).[14]

An incipient literature has started to examine the relationship between termination rates and equilibrium prices in an environment with call externalities and RPP regimes. The papers closest to ours are JLT, Cambini and Valletti (2008), López (2011) and Hoernig (2016).[15] Our model is an oligopoly extension of JLT and assumes passive rather than responsive expectations. JLT use the vanishing noise criterion to address the issue of multiplicity and notice that efficiency cannot be attained because of connectivity breakdown. López (2011) obtains the same result for an asymmetric duopoly with non-vanishing noise. Cambini and Valletti (2008) resolve the problem of connectivity breakdown by adding call propagation to the JLT model. Moreover, they show that when firms bargain about a reciprocal termination charge, agreement will be reached on the efficient one. We tackle the connectivity breakdown by considering an oligopoly[16] and show that firms will not agree on the efficient charge when consumers have passive expectations. An important further contribution of our paper is of course that we characterize all equilibria and do not limit the attention to equilibria that are robust to adding vanishing noise. This allows us to compare the European and US telecommunication markets in terms of efficiency and profitability.

Our paper proceeds as follows. Section 2 generalizes the model of JLT by considering a Logit formulation with any number of networks. It also defines the concept of passive expectations. In section 3 we derive the set of equilibria and discuss its properties. Section 4 compares the European scenario of CPP with cost-based termination with the US scenario of RPP and Bill and Keep arrangements. Section 5 concludes. Appendix A discusses two alternative equilibrium selection theories while Appendix B explains the role of passive expectations. Appendix C contains all proofs.

[14]These papers have in common that they add a realistic feature (expansion in the mobile market and uniform FTM and MTM termination rates, unbalanced calling patterns, elastic subscription for those consumers who only enjoy receiving calls, and income effects, respectively) to the theoretical model and conclude that, for some parameter range, firms prefer termination charges above cost. The first three articles conclude that efficient termination charges are above cost so that the need for regulation is mitigated.

[15]Other related papers in this literature include Kim and Lim (2001), Hermalin and Katz (2001, 2004, 2011), Degraba (2003), Hahn (2003), Laffont et al. (2003), Berger (2004, 2005).

[16] Hoernig (2016), using responsive expectations, also shows that breakdown only occurs for the duopoly case.

2 The model

We consider a general model of $n \geq 2$ network operators. The n network operators have complete coverage and compete for a continuum of consumers of unit mass.

Timing. We assume that the terms of interconnection are negotiated or established by a regulator first. Then, for a given reciprocal[17] access charge $a \geq 0$, the timing of the game is the following:

1. Consumers form expectations about the number of subscribers of each network i: α_i^e.
2. Firms take these expectations as given and choose simultaneously retail tariffs.
3. Consumers make rational subscription and consumption decisions, given their expectations and given the networks' tariffs.

Consumers thus hold passive expectations which do not change when a firm changes a price. Realized market share α_i is a function of prices and consumer expectations. Self-fulfilling expectations imply that at a symmetric equilibrium $\alpha_i^e = \alpha_i = 1/n$.[18]

Cost structure. The fixed cost to serve each subscriber is f, whereas c_O and c_T denote the marginal cost of providing a telephone call borne by the originating and terminating networks. The marginal cost of an on-net call is $c = c_O + c_T$. Let us denote the termination mark-up by

$$m = a - c_T.$$

The perceived marginal cost of an off-net call for the originating network is the true cost c for on-net calls, augmented by the termination mark-up for the off-net calls: $c_O + a = c + m$. The perceived marginal cost of an off-net call for the terminating network is $c_T - a = -m$.

Pricing. Each firm $i \in N = \{1, 2, \ldots, n\}$ charges a tariff $T_i = (F_i, p_i, r_i, \hat{p}_i, \hat{r}_i)$, consisting of a fixed fee (F_i), per-unit call and reception charges for on-net traffic (p_i and r_i) and per-unit call and reception charges for off-net traffic ($\hat{p}_i$ and $\hat{r}_i$).[19] We restrict all prices to be non-negative.

[17] Reciprocity means that a network pays as much for termination of a call on the rival network as it receives for completing a call originated on the rival network.

[18] If consumers form expectations after tariffs are observed (as assumed in JLT), consumers' expectations are responsive because they would depend on the prices chosen. In Section B we compare the outcomes under passive and responsive expectations.

[19] When $n \geq 3$, it would be even more general to allow each firm to set different prices for off-net traffic depending on which network is being called or is calling. However, since attention will be restricted to symmetric equilibria we lose nothing from imposing that there cannot be discrimination between the prices set for traffic terminating or originating at different rival networks. This reduces the burden of notation.

Individual demand. Subscribers obtain positive utility from making and receiving calls. The caller's utility from making a call of length q minutes is $u(q)$, whereas the receiver's is $\widetilde{u}(q)$ from receiving a call of that length. $u(\cdot)$ and $\widetilde{u}(\cdot)$ are twice continuously differentiable, increasing and concave. For tractability, we assume that

$$\widetilde{u}(q) = \beta u(q) \quad \text{with } 0 < \beta < 1,$$

where β measures the strength of the call externality. The caller's demand function is given by $u'(q(p)) = p$, whereas the receiver's demand function is given by $\widetilde{u}'(q(r)) = r$. We consider the case in which both callers and receivers can hang up. Defining $D(p,r) = q(\max\{p, r/\beta\})$, the length of an on-net call is $D(p_i, r_i)$, whereas the length of an off-net call is $D(\hat{p}_i, \hat{r}_j)$ (for $i \in N$ and $j \in N \setminus \{i\}$). We will denote $U(p,r) = u(D(p,r))$ and $\widetilde{U}(p,r) = \beta u(D(p,r))$.

Market shares. We are interested in allowing for industry structure with more than two firms. We will use the Logit formulation.[20] We make the standard assumption of a balanced calling pattern, which means that the fraction of calls from a given subscriber of a given network and completed on another given (including the same) network is equal to the fraction of consumers subscribing to the terminating network.[21] Let w_i denote the expected value of subscribing to network i. That is,

$$\begin{aligned} w_i \;=\; & \alpha_i^e(U(p_i,r_i) + \widetilde{U}(p_i,r_i) - (p_i + r_i)D(p_i,r_i)) - F_i \\ & + \sum_{j \neq i} \alpha_j^e(U(\hat{p}_i,\hat{r}_j) - \hat{p}_i D(\hat{p}_i,\hat{r}_j)) \\ & + \sum_{j \neq i} \alpha_j^e(\widetilde{U}(\hat{p}_j,\hat{r}_i) - \hat{r}_i D(\hat{p}_j,\hat{r}_i)) \end{aligned}$$

The first line corresponds to the utility from placing and receiving on-net calls, the second to the utility from placing off-net call and the third to the utility from receiving off-net calls. Consumers have idiosyncratic tastes for each operator. We add a random noise term ε_i and define $U_i = w_i + \mu\varepsilon_i$. The parameter $\mu > 0$ reflects the degree of product differentiation in a Logit model. A high value of μ implies that most of the value is determined by a random draw so that competition between the firms is rather weak. The noise terms ε_k are random variables of zero mean and variance $\pi^2/6$, identically and independently double exponentially distributed. These terms reflect consumers' preference for one good over another (they are known to the consumer but are unobserved by the firms). A consumer will subscribe to network $i \in N$ if and only if $U_i > U_j$ for

[20] See Anderson and De Palma (1992) and Anderson et al. (1992) for more details about the Logit model.

[21] Dessein (2003, 2004) examines how unbalanced calling patterns between different customer types affect retail competition when network operators compete in the presence of the caller-pays regime.

all $j \in N \setminus \{i\}$. The probability of subscribing to network i is denoted by α_i. The probabilities (or equivalently, market shares) are given by

$$\alpha_i = \frac{\exp[w_i/\mu]}{\sum_{k=1}^{n} \exp[w_k/\mu]}. \tag{1}$$

Consumer Surplus. Consumer surplus in the Logit model has been derived by Small and Rosen (1981) as (up to a constant)

$$CS = \mu \ln \left(\sum_{k=1}^{n} \exp(w_k/\mu) \right) = w + \mu \ln n, \tag{2}$$

where the last equation holds in case of a symmetric solution where each network offers surplus $w_i = w$.

Profit. Fixing tariffs of firms $j \neq i$ at $(F^*, p^*, r^*, \hat{p}^*, \hat{r}^*)$, firm i's profit is given by

$$\begin{aligned} \pi_i \quad = \quad & \alpha_i \Big[\alpha_i (p_i + r_i - c) D(p_i, r_i) + F_i - f \Big] + \\ & \alpha_i \Big[(1 - \alpha_i)(\hat{p}_i - c - m) D(\hat{p}_i, \hat{r}^*) + (1 - \alpha_i)(\hat{r}_i + m) D(\hat{p}^*, \hat{r}_i) \Big] \end{aligned} \tag{3}$$

3 Symmetric equilibrium analysis

In this Section we will characterize all symmetric equilibria without connectivity breakdown. It is straightforward to see that there always exists an equilibrium without any off-net traffic. All firms charging infinite call and reception prices for off-net traffic constitutes part of an equilibrium.[22] There may also exist equilibria in which off-net traffic is choked off by infinite reception charges only.

We will, however, focus on symmetric equilibria without connectivity breakdown so that off-net call volume is strictly positive. Of course, we do take into account that each network could potentially choke off inbound or outbound off-net traffic. That is, we need to make sure that firms have no incentives to do so.

Firms will set on-net call and reception charges so as to maximize the utility obtained from on-net traffic by internalizing the call externality. Optimality requires that the volume of on-net traffic q satisfies $(1 + \beta)u'(q) = c$. This can be obtained by setting prices (p^*, r^*) where

$$p^* = \frac{c}{1 + \beta}, \quad r^* = \frac{\beta c}{1 + \beta}. \tag{4}$$

Of course, the optimal volume can also be obtained by setting on-net prices (p^*, r_i) with

[22]On-net prices will be set efficiently and fixed fees are used to fight for market share. Firms will not make any profit on call and reception services and just choose fixed fee to maximize $\pi = \alpha(F - f)$. In a symmetric equilibrium this yields $F^* = f + n\mu/(n - 1)$ and profit $\pi^* = \mu/(n - 1)$.

$r_i < r^*$ or (p_i, r^*) with $p_i < p^*$. We will assume (for now and without loss of generality) that prices (p^*, r^*) are chosen.[23] Note that $p^* + r^* = c$ so that no profit is obtained from on-net traffic (except for the fixed fee that is levied on subscribers). Note that at prices p^* and r^* caller and receiver share the cost of a call in the proportion they benefit from it. These prices will also play a role when we discuss efficient off-net prices.

Determining the equilibrium prices for off-net traffic from and to network i is more complicated because these prices $\hat{p}_i$ and $\hat{r}_i$ not only affect own subscribers, but also consumers on other networks through the call externality: if a reduction in $\hat{p}_i$ raises call volume (caller determined volume), consumers on rival networks will receive (and pay for) more calls; if a reduction in $\hat{r}_i$ raises call volume (receiver determined volume), consumers on rival networks will place (and pay for) more calls. Finally, there are also equilibria in which neither a reduction in $\hat{p}_i$ nor in $\hat{r}_i$ raises call volume. We analyze these equilibria in turn in the next subsections.

3.1 Call volume determined by caller

We first restrict attention to equilibria in which the caller determines the volume of calls. In particular, we look for a symmetric equilibrium so that expected market shares equal $\alpha_i^e = 1/n$ for all i. Since callers determine the volume of calls, at the equilibrium the first-order condition with respect to the off-net call price must hold. This yields the following necessary condition:

$$\hat{p}^* = \frac{(n-1)(c+m) - \hat{r}^*}{n-1-\beta}. \tag{5}$$

Clearly, the off-net call price increases with the cost of originating an off-net call, $c + m$, and decreases with the off-net reception charge. Intuitively, when the volume of calls from one network to another increases, the receiver's surplus increases because of the extra calls, but the receiver also has to pay for the extra calls. This is a pecuniary externality that decreases the perceived marginal cost (see JLT) and allows the originating network to increase its fixed fee with no loss in market share. Additionally, at equilibrium we must have that

$$\hat{p}^* \geq \hat{r}^*/\beta, \tag{6}$$

otherwise the receiver would hang up first. The following Proposition characterizes such an equilibrium:

Proposition 1. *[Caller determined volume]*

[i] If $(p^*, r^*, \hat{p}^*, \hat{r}^*, F^*)$ is a symmetric equilibrium in which the caller determines the

[23] If there would be vanishing noise in the receiver's utility then the unique optimal prices would converge to (p^*, r^*). This is formally shown in the proof of Proposition 6 below.

call volume, then $\hat{p}^*$, $\hat{r}^*$ and F^* must satisfy necessary conditions (5), (6),

$$\hat{p}^* \geq -m/\beta \tag{7}$$

and

$$F^* = f + \frac{n\mu}{n-1} - \frac{n-2}{n}(\hat{p}^* + \hat{r}^* - c)q(\hat{p}^*). \tag{8}$$

[ii] If $\hat{p}^*$, $\hat{r}^*$ and F^* satisfy conditions (5)-(8), $(n-1)\beta > 1$ and μ is sufficiently high, then $(p^*, r^*, \hat{p}^*, \hat{r}^*, F^*)$ is a symmetric equilibrium in which the caller determines the call volume, and equilibrium profit equals

$$\pi^* = \frac{\mu}{n-1} + \frac{1}{n^2}(\hat{p}^* + \hat{r}^* - c)q(\hat{p}^*). \tag{9}$$

While condition (6) ensures that the caller hangs up first, the firm must have no incentives to raise the reception price for off-net calls above $\beta\hat{p}^*$. In the proof of Proposition 1 we show that when $\beta\hat{p}^* < -m$ or $\beta\hat{p}^* = -m$ and $(n-1)\beta < 1$, the firm has an incentive to deviate by setting reception price marginally above $\beta\hat{p}^*$. Conversely, when $\beta\hat{p}^* \geq -m$, $(n-1)\beta > 1$ and product differentiation parameter μ is sufficiently high, then the firm has no incentive to deviate and $(p^*, r^*, \hat{p}^*, \hat{r}^*, F^*)$ is in fact an equilibrium.

Proposition 1 unveils that the number of firms is relevant when checking whether a firm has an incentive to create connectivity breakdown by raising the off-net reception price to infinity. If the number of firms is high (so that $\beta > 1/(n-1)$) there is no incentive to do this. The intuition is clear: if network i provokes connectivity breakdown this will affect the subscribers of other networks only with respect to the calls made to subscribers of network i. This is only a (small) fraction $1/(n-1)$ of all off-net calls made. On the other hand, the subscribers of network i will not be able to receive any off-net call. As long as $\beta > 1/(n-1)$ connectivity breakdown hurts subscribers from network i more than those of rival networks.

If, on the other hand, $(n-1)\beta < 1$, provoking connectivity breakdown by unilaterally setting reception charge equal to infinity may (but need not) be a profitable deviation from the proposed price schedule. In particular, if $n = 2$ the sufficient condition in part [ii] of Proposition 1 is never satisfied and one needs to check carefully whether provoking connectivity breakdown is a profitable deviation from the proposed prices. The profitability of such a deviation depends on the particular equilibrium candidate under consideration, the termination charge, and the strength of the call externality. In particular, provoking connectivity breakdown will be profitable for low values of the call externality, but not for very high ones.

As an example, consider a duopoly and suppose that $-\beta c/(1+\beta) < m \leq 0$. Consider the candidate equilibrium off-net prices $(\hat{p}, \hat{r}) = ((c+2m)/(1-\beta), -m)$. This is the candidate equilibrium proposed in JLT, Prop. 9[i]. This candidate equilibrium with caller determined volume results in asymptotic connectivity breakdown as $\beta \to 1$. Unlike them, we can study whether such prices are actually an equilibrium when $\beta < 1$ by checking whether provoking connectivity breakdown by raising the reception charge is profitable. Note first that no profit is made from receiving off-net calls because $\hat{r} + m = 0$. By deviating to $\hat{r}_1 = \infty$ network 1 hurts subscribers on the rival network more than its own subscribers when the call externality is weak. As a result, it can raise the fixed fee to its own subscribers such that it still attracts half of the market. This clearly raises the profit of network 1, and the candidate equilibrium is then not an actual equilibrium. On the other hand, when the call externality is relatively strong then provoking connectivity breakdown (by raising the reception charge) hurts own subscribers more than the rival's subscribers, because the caller on a rival network pays more for off-net calls than the receiver on the own network does, while the utility obtained from such calls is almost the same for both parties.

3.2 Call volume determined by receiver

We look for a symmetric equilibrium in which receivers determine the volume of off-net calls – consequently, the first-order condition with respect to $\hat{r}$ must hold:

$$\hat{r}^* = \frac{\beta((n-1)m + \hat{p}^*)}{1-(n-1)\beta}. \tag{10}$$

The off-net reception charge increases with the cost of receiving an off-net call, m, and with the off-net call price, because of the pecuniary externality. Of course, condition (10) is not sufficient: not only we need

$$\hat{r}^* \geq \beta \hat{p}^*, \tag{11}$$

so that receivers hang up first, but also that firms have no incentives to raise their off-net call price above $\hat{r}^*/\beta$. Let

$$\overline{m}^R = -\frac{((n-1)\beta - 1)c}{n-2+(n-1)\beta} < 0,$$

then we have:

Proposition 2. *[Receiver determined volume]*

[i] If $(n-1)\beta < 1$ or $m > \overline{m}^R$, there exists no symmetric equilibrium in which the receiver determines the call volume.

[ii] If $(n-1)\beta > 1$, $m \leq \overline{m}^R$, and μ is sufficiently high, then $(p^*, r^*, \hat{p}^*, \hat{r}^*, F^*)$ is a symmetric equilibrium in which the receiver determines the call volume if and only if $\hat{p}^*$, $\hat{r}^*$ and F^* satisfy necessary conditions (10), (11),

$$\hat{r}^* \geq \beta(c+m) \tag{12}$$

and

$$F^* = f + \frac{n\mu}{n-1} - \frac{n-2}{n}(\hat{p}^* + \hat{r}^* - c)q(\hat{r}^*/\beta). \tag{13}$$

Moreover, equilibrium profit then equals

$$\pi^* = \frac{\mu}{n-1} + \frac{1}{n^2}(\hat{p}^* + \hat{r}^* - c)q(\hat{r}^*/\beta).$$

When $(n-1)\beta < 1$ the first-order condition fails to hold, and firms set the off-net reception charge either at $\beta\hat{p}^*$ or ∞ (thereby creating a connectivity breakdown). Condition (12) ensures that a marginal deviation by setting $\hat{p}^*$ slightly above $\hat{r}^*/\beta$ is not profitable (i.e., $\partial\pi^*/\partial\hat{p}_i < 0$ at $\hat{p}_i = \hat{r}^*/\beta$); large deviations are also unprofitable when μ is sufficiently high. Finally, (10) together with (12) yield the condition on the termination mark-up. To put it simply, when $m > \overline{m}^R$ there does not exist a symmetric equilibrium in which the receiver determines the call volume because the two conditions are then incompatible.

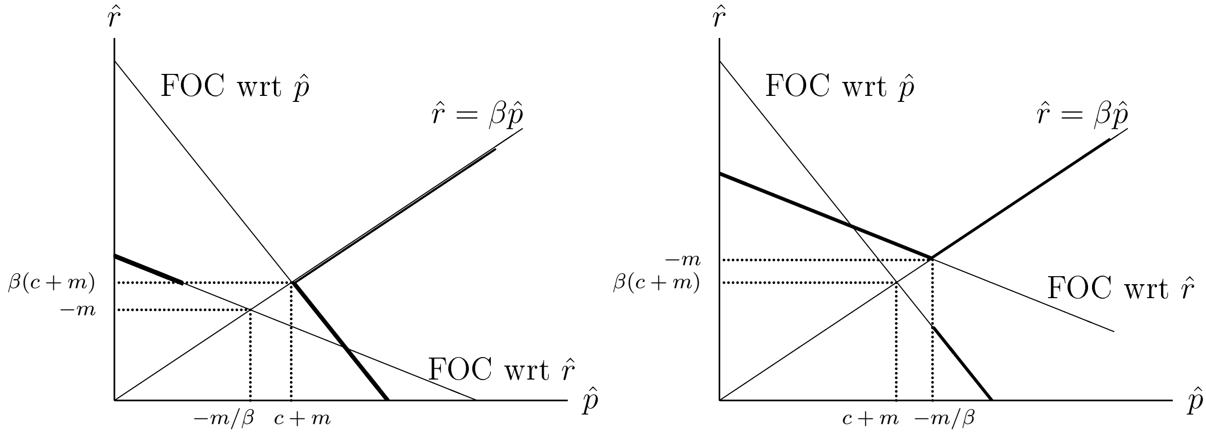
3.3 Other equilibria

Finally, we discuss symmetric equilibria that are not characterized by binding first-order conditions. Such equilibria have $\hat{r} = \beta\hat{p}$, so that neither a reduction in $\hat{p}_i$ nor in $\hat{r}_i$ increase call volume from or to operator i. Of course, an increase in one of these prices does reduce call volume. Hence, necessary conditions at such equilibria are that $\partial\pi_i/\partial\hat{r}_i \leq 0$ and that $\partial\pi_i/\partial\hat{p}_i \leq 0$. The analysis from the previous two subsections shows that both conditions are satisfied when $\hat{p} \geq \max\{c+m, -m/\beta\}$.

3.4 Equilibrium Set and Duality

Figure 1 illustrates the set of equilibrium off-net usage prices for a fixed m when $(n-1)\beta > 1$, $m \leq \overline{m}^R$ and μ sufficiently high. Note the special role played by $\overline{m} = -\beta c/(1+\beta)$. In particular, $-m/\beta < c+m$ if and only if $m > \bar{m}$.

Propositions 1 and 2 establish for *a particular* termination mark-up m the set of equilibria in which callers and receivers determine call volume. A continuum of equilibria exists for each m. Profit and welfare properties of the different equilibria depend on m only through off-net prices $\hat{p}$ and $\hat{r}$. Performing policy analysis and recommending termination rates would require to know which equilibrium will be played. There is no unique obvious way of selecting one equilibrium. Before proposing possible equilibrium selection criteria, we first investigate the set of all possible combinations of off-net call and reception charges that can prevail in an equilibrium for *some* termination mark-up. This will allow us to analyze whether and how efficiency can be obtained. Moreover, it will give some insight in the overall profitability of the different equilibria. Finally, it



(a) $m > \bar{m}$

(b) $m < \bar{m}$

Figure 1: Equilibrium off-net prices for a fixed termination mark-up m.

will allow us to clarify the similarity, or better, duality between caller determined and receiver determined call volume equilibria.

Let us denote by E^C (resp. E^R) the set of off-net prices $(\hat{p}, \hat{r})$ in an equilibrium where callers (resp. receivers) determine the call volume. Because on-net prices are the same in all equilibria and equilibrium fixed fees are a function of off-net prices, we can identify equilibrium off-net prices with equilibria.

Let $(\hat{p}, \hat{r}) \in E^C$. Then, from Eq. (5), the termination mark-up must equal $m^C(\hat{p}, \hat{r})$ where

$$m^C(p, r) = \frac{(n-1-\beta)p + r}{n-1} - c.$$

Furthermore, necessary conditions are that $\beta\hat{p} \geq \hat{r} \geq 0$ and $\beta\hat{p} \geq -m^C(\hat{p}, \hat{r})$. Since we do not allow for negative termination charges a final necessary condition is that $m^C(\hat{p}, \hat{r}) \geq -c_T$. From Proposition 1 it follows that any pair $(\hat{p}, \hat{r})$ that satisfies the above conditions is part of an equilibrium when $(n-1)\beta > 1$ and μ is sufficiently high.

Let now $(\hat{p}, \hat{r}) \in E^R$. In this case, from Eq. (10), the termination mark-up must equal $m^R(\hat{p}, \hat{r})$ where

$$m^R(p, r) = \frac{((1-(n-1)\beta)r)/\beta - p}{n-1}.$$

In addition, necessary conditions are $\hat{r} \geq \beta\hat{p} \geq 0$ and $\hat{r} \geq \beta(c + m^R(\hat{p}, \hat{r}))$. Because of negative termination charges are excluded from the analysis, we have as a final necessary condition that $m^R(\hat{p}, \hat{r}) \geq -c_T$. From Proposition 2 it follows that any pair $(\hat{p}, \hat{r})$ that satisfies the above conditions is part of an equilibrium when $(n-1)\beta > 1$ and μ is sufficiently high.

Thus we obtain the following corollary.

Corollary 1. *Assume* $(n-1)\beta > 1$ *and* μ *sufficiently high. Then the set of equilibrium*

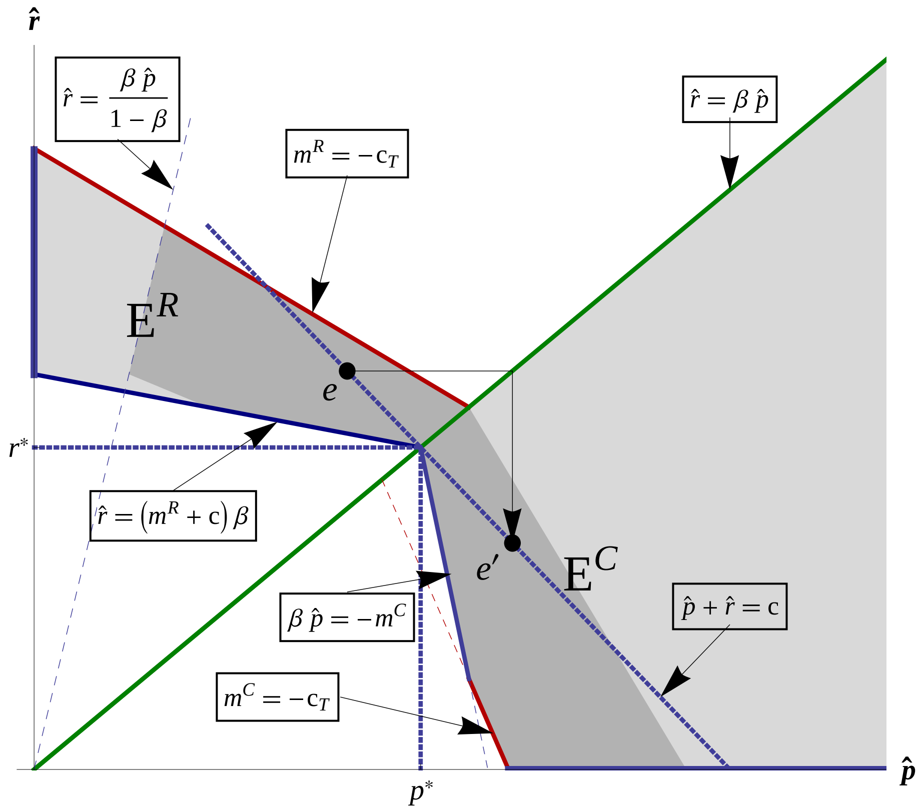


Figure 2: Off-net equilibrium prices in which caller (E^C) or receiver (E^R) determines call volume. For equilibrium prices in the dark shaded areas there exists a dual equilibrium in which the other party determines call volume.

off-net prices in an equilibrium with caller determined volume is

$$E^C = \left\{(\hat{p}, \hat{r}) \in \Re^2_+ \quad : \quad m^C(\hat{p}, \hat{r}) \geq -c_T \text{ and } \beta\hat{p} \geq \max\{\hat{r}, -m^C(\hat{p}, \hat{r})\}\right\}$$

and the set of equilibrium off-net prices in an equilibrium with receiver determined volume is

$$E^R = \left\{(\hat{p}, \hat{r}) \in \Re^2_+ \quad : \quad m^R(\hat{p}, \hat{r}) \geq -c_T \text{ and } \hat{r} \geq \max\{\beta\hat{p}, \beta[c + m^R(\hat{p}, \hat{r})]\}\right\}.$$

Recall that p^* and r^* as defined in Eq. (4) are the efficient (on-net) prices. Note that $m^C(p^*, r^*) = m^R(p^*, r^*) = -\beta c/(1+\beta) = \overline{m}$ so that $(p^*, r^*) \in E^C \cap E^R$ if and only if $\beta c/(1+\beta) \leq c_T$, which we will assume to hold. Note that a sufficient condition for this is that $c_T = c/2$. The sets E^C and E^R are illustrated in Figure 2.

Because the lines defined by $\beta\hat{p} = -m^C(\hat{p}, \hat{r})$ and $\hat{r} = \beta(c + m^R(\hat{p}, \hat{r}))$ are downward sloping and go through the point (p^*, r^*), it follows that any pair of off-net equilibrium prices $(\hat{p}, \hat{r}) \neq (p^*, r^*)$ leads to inefficiently low call volume. Hence, even though any off-net prices $\hat{p} = p^*$ and $\hat{r} < \beta p^*$ (or $\hat{r} = r^*$ and $\hat{p} < r^*/\beta$) would yield efficient off-net call volume, none of these combinations can prevail in an equilibrium for any termination mark-up. In particular, any equilibrium with zero charge for reception (as in Europe) must be inefficient and any equilibrium with $\hat{p} = \hat{r}$ (as in the US) must be inefficient.

Let us now consider the profitability of the different equilibrium outcomes. Consider an equilibrium in which the caller determines call volume with $\beta\hat{p} > \hat{r}$ and $\hat{p} + \hat{r} > c$. It is immediate that the equilibrium $(\hat{p} - \varepsilon, \hat{r} + \varepsilon)$ yields higher profit for small $\varepsilon > 0$, because the strictly positive profit margin remains the same while call volume increases. Hence, the maximum profit to be obtained in any equilibrium in which callers determine call volume is obtained for some equilibrium with $\beta\hat{p} = \hat{r}$. The same argument holds for the optimal payoff to be obtained in any equilibrium in which the receiver determines the call volume. Hence, the profit maximizing equilibrium must have $\hat{r}^{\pi} = \beta\hat{p}^{\pi}$ where $\hat{p}^{\pi} = \arg\max_p((1+\beta)p - c)q(p)$. In case of iso-elastic call demand, $q(p) = p^{-\eta}$, the maximum is obtained when $\hat{p} = p^M/(1+\beta)$, where $p^M = \arg\max_p\{(p-c)q(p)\} = \eta c/(\eta - 1)$ denotes the monopoly (call) price. Total price (i.e., $\hat{p}^{\pi} + \hat{r}^{\pi}$) is equal to monopoly price but call volume is strictly higher than monopoly call volume. Note that $p^{\pi} > p^*$. The maximal profit can be achieved in a caller determined volume equilibrium when the termination mark-up is equal to $m^C(\hat{p}^{\pi}, \hat{r}^{\pi}) > \overline{m}$, and possibly also in a receiver determined volume equilibrium when the termination mark-up equals $m^R(\hat{p}^{\pi}, \hat{r}^{\pi}) < \overline{m}$. However, the latter may be below $-c_T$ which would require a negative termination charge, that is, a payment from the terminating network to the originating network which is not allowed for in our model, to avoid (as explained below) arbitrage opportunities. We summarize our results.

Proposition 3.

[i] The total welfare maximizing equilibrium is obtained for off-net prices (p^*, r^*) and achieves full efficiency. Any other equilibrium exhibits inefficiently low call volume. Efficiency can only be obtained when $m = \overline{m}$.

[ii] The profit maximizing equilibrium is obtained for off-net prices $(\hat{p}^{\pi}, \hat{r}^{\pi})$ with

$$\hat{r}^{\pi} = \beta\hat{p}^{\pi} \text{ and } \hat{p}^{\pi} = \arg\max_p((1+\beta)p - c)q(p).$$

Maximum profit can be obtained when $m = m^C(p^{\pi}, r^{\pi}) > \overline{m}$ and by $m = m^R(p^{\pi}, r^{\pi}) < \overline{m}$ (if $m^R(p^{\pi}, r^{\pi}) \geq -c_T$).

Duality. It is not a coincidence that the maximal profit can possibly be obtained by two different equilibria. It is obvious that any $(\hat{p}, \hat{r}) \in E^R \cap E^C \setminus \{(p^*, r^*)\}$ can be obtained both in an equilibrium with call volume determined by the caller (namely when $m = m^C(\hat{p}, \hat{r})$) and in an equilibrium with call volume determined by the receiver (namely when $m = m^R(\hat{p}, \hat{r})$). However, the idea that what can be obtained in an equilibrium in which the receiver determines volume can also be obtained in an equilibrium in which the caller determines volume, and vice versa, holds more generally. To be more precise, let us say that $e' = (\hat{p}', \hat{r}') \in E^C$ is the *dual* of $e = (\hat{p}, \hat{r}) \in E^R$ if call volume and profit in e are the same as call volume and profit in e'. Dual equilibria are equally efficient and yield the same profit, and thus also yield exactly the same consumer surplus. The only differences lie in the identity of the person who determines the volume of the call and in the required termination rate to support these prices as an equilibrium. It is easy to construct for each $e \in E^R$ the unique candidate dual, e'. Namely, to have equal call volume it must be that $\hat{p}' = \hat{r}/\beta$. To have moreover the same profit, it must be that $\hat{r}' = \hat{p} + \hat{r} - \hat{p}'$. Of course, in general the constructed e' may not lie within E^C, as it may not satisfy some of the linear constraints that characterize E^R. For example, it is straightforward to show that $\hat{r}' \geq 0$ requires that $\hat{r}/\hat{p} \leq \beta/(1-\beta)$. Also, the condition $\beta\hat{p}' \geq -m^C(\hat{p}', \hat{r}')$ is equivalent to $\hat{r} \geq \beta[c + m^R(\hat{p}, \hat{r})]$.

Proposition 4 (Duality property)**.** *There exist non-empty subsets $\tilde{E}^R \subset E^R$ and $\tilde{E}^C \subset E^C$ such that for any $e \in \tilde{E}^R$ there exists its dual $e' \in E^C$, where call volume, profit and consumer surplus are the same. Similarly, for any $e' \in \tilde{E}^C$ there exists its dual $e \in E^R$, where call volume, profit and consumer surplus are the same. Moreover, equilibria outside $\tilde{E}^R$ and $\tilde{E}^C$ do not have a dual equilibrium.*

Figure 2 shows the construction of the dual equilibrium. The dark shaded areas indicate the subsets of E^R and E^C that have dual equilibria. The duality property illustrates the idea that there is no fundamental difference between origination and termination or between call and reception charges. Only the restriction to non-negative call, reception and termination prices limits the duality property to a subset of equilibria. Of course, negative prices may lead firms or consumers to engage in arbitrage opportunities: For example, a sufficiently negative termination charge (or positive origination charge) may give firms perverse incentives to place a huge amount of off-net calls to collect "origination rents".

4 Comparing outcomes in Europe and US

We have shown that a multiplicity of equilibria exists for any given termination rate. Without addressing this coordination problem it is hard, if not impossible, to do policy analysis or cross-country comparisons. Namely, the termination mark-ups that maximize producer or total welfare depend on which of the equilibria networks play. JLT and related subsequent papers select a unique equilibrium (candidate) by adding vanishing noise to the marginal utilities of receivers. The underlying assumption of small random utility shocks has certainly some appeal and the characterization of the unique equilibrium candidate is rather elegant. However, the unique theoretical prediction does not always resemble actual outcomes in practice, especially when termination rates are low.[24]

[24] The theory predicts in general *different* and *positive* prices for receiving and placing calls when termination rates are below cost, and this is not usually observed in practice. For completeness and comparison purposes, we discuss this equilibrium selection theory in Appendix A. We there also discuss the selection criterion of firms coordinating on the joint profit maximizing equilibrium.

We propose two alternative selection theories based on actual observations. First, in most countries around the world, including all European countries, consumers are not charged for receiving (domestic) calls.[25] That is, in these countries firms seem to play the equilibrium with zero reception charges ($r = \hat{r} = 0$). We call this the (voluntary) CPP equilibrium. Second, in other countries where consumers are charged for reception, including the US, the prices for receiving and placing calls are usually equal.[26] That is, in these countries firms seem to play the equilibrium in which $p = r$ and $\hat{p} = \hat{r}$, which we denote by RPP*. These alternative equilibrium selection theories are less ad hoc than they may seem. In the early stages of mobile communication in Europe, mobile penetration was low and termination on mobile networks was not regulated, whereas termination on the incumbent fixed network was regulated at a low level. Most calls were from and to fixed lines, so mobile operators had strong incentives to (1) set high termination rates and (2) not charge their subscribers for receiving calls. In the US, on the other hand, fixed and mobile operators were required to interconnect at very low ($0.0007) reciprocal termination rates. Mobile operators therefore had incentives to charge receivers.

We now consider the voluntary CPP and RPP* equilibria and characterize private and social welfare maximizing termination rates for both. We then use the results to provide a comparison of the European and US telecommunication markets. In order to avoid potential problems of equilibrium existence, we will assume from now on that $(n-1)\beta > 1$ and that the product differentiation parameter μ is sufficiently large. These conditions are sufficient for the existence of the equilibria characterized in Proposition 1. The first condition is also a necessary condition for equilibria to exist in which the receiver determines the volume (see Proposition 2).

4.1 CPP

If firms play an equilibrium in which receiving calls is free of charge, they must play the equilibrium $(p, r, \hat{p}^*, \hat{r}^*, F^{**})$ where $r = \hat{r} = 0$, $p = p^*$ and, from Proposition 1,

$$\hat{p}^* = \frac{(n-1)(c+m)}{n-1-\beta}$$

and

$$F^{**} = f + \frac{n\mu}{n-1} - \frac{n-2}{n}(\hat{p}^* - c)q(\hat{p}^*) + \frac{1}{n}r^* q(r^*/\beta)$$

Note that because on-net reception price is now equal to zero, the new equilibrium fixed fee includes the payment that in the equilibrium described in Proposition 1 was collected through positive on-net reception charges. Equilibrium profit is still given by

[25] Consumers do pay roaming charges for receiving (and placing) international calls.

[26] For example, when consumers purchase a bundle of X minutes per month, both placing and receiving a one minute call reduces the budget of remaining minutes by one.

(9) (with $\hat{r}^* = 0$).[27]

Recall from Proposition 1 that a necessary condition for this equilibrium to exist is that $\beta\hat{p}^* \geq -m$, which is equivalent to $m \geq \underline{m}^{\text{CPP}}$, where

$$\underline{m}^{\text{CPP}} = \frac{-\beta c}{\beta + 1 - \frac{\beta}{n-1}} < 0.$$

In particular, when termination mark-ups are positive (as they have been in Europe), the CPP equilibrium exists. Only when termination rates are sufficiently below cost, the CPP equilibrium may cease to exist.

The termination mark-up that maximizes firms' profits is the one that maximizes $(\hat{p}^* - c)q(\hat{p}^*)$. That is, firms prefer termination mark-up that yields off-net price equal to monopoly price p^M. Hence, the termination mark-up that maximizes firms' profits equals

$$m^{\pi} = \frac{n-1-\beta}{n-1}p^M - c.$$

Observe that $m^{\pi} > \underline{m}^{\text{CPP}}$ so that this profit maximizing equilibrium indeed exists.

The socially optimal termination mark-up would be the one that achieves the efficient call volume, *i.e.* such that $\hat{p}^* = p^*$. Hence, the optimal termination mark-up would be

$$m^W_{\text{CPP}} = \left(\frac{-\beta c}{1+\beta}\right)\left(\frac{n}{n-1}\right) < 0.$$

However, it is easily established that $m^W_{\text{CPP}} < \underline{m}^{\text{CPP}}$. This means that regulating termination charges at efficient levels is incompatible with a voluntary CPP equilibrium.[28] This was of course already clear from Proposition 3, where we saw that efficiency can only be obtained with off-net prices equal to (p^*, r^*). Were regulators to impose m^W_{CPP} hoping to achieve efficiency, operators would be forced to play an equilibrium in which consumers are charged a positive price for receiving calls. Alternatively, regulators could impose the lowest termination mark-up compatible with voluntary CPP (i.e., $\underline{m}^{\text{CPP}}$), and accept some inefficiency.[29]

4.2 RPP*

From Proposition 2 we know that an equilibrium with equal charge for placing and receiving calls can only exist when the termination mark-up is sufficiently low. In particular,

[27] When $n = 2$, the off-net call price converges to infinity as the call externality β tends to one: the equilibrium exhibits asymptotic connectivity breakdown (see JLT and López, 2011). This does not occur when $n > 2$, which is implied by our assumption $(n-1)\beta > 1$.

[28] If firms are prevented from the option to charge reception (perhaps by law), then efficiency can be achieved in a CPP equilibrium (see Berger, 2005).

[29] If $\underline{m}^{\text{CPP}} < -c_T$, the best regulators can do is to impose Bill and Keep.

the termination mark-up must be negative. Such an equilibrium[30] must have

$$\hat{p} = \hat{r} = -\frac{\beta(n-1)m}{n\beta - 1}. \tag{14}$$

The necessary condition $\hat{r} \geq \beta(c+m)$ is satisfied if and only if $m \leq \overline{m}^{\text{RPP}}$, where

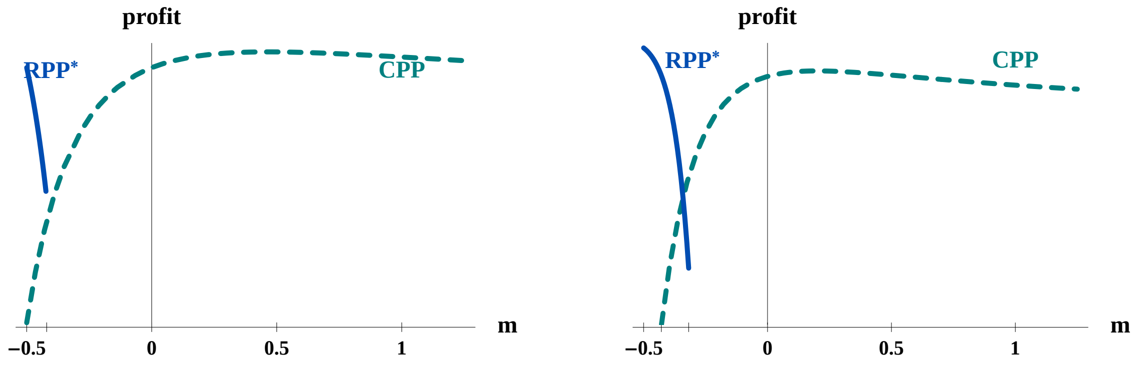
$$\overline{m}^{\text{RPP}} = \frac{c(1-n\beta)}{n-2+n\beta}.$$

The social welfare maximizing termination mark-up would be the one that makes $\hat{r}$ equal to $\beta c/(\beta+1)$, so that $m^W_{\text{RPP}} = c(1-n\beta)/((1+\beta)(n-1))$. However, $m^W_{\text{RPP}} > \overline{m}^{\text{RPP}}$. Regulating termination charges at efficient levels is thus incompatible with an RPP* equilibrium. As in the case of CPP, this was already clear from Proposition 3 where we saw that the only efficient equilibrium off-net prices satisfy $r^* = \beta p^* < p^*$. Alternatively, regulators could try to impose the highest termination mark-up compatible with RPP* (*i.e.*, $\overline{m}^{\text{RPP}}$), and accept some inefficiency. However, even this may be problematic because it may be in the interest of firms to agree on lower termination rates, such as Bill and Keep, which may lead to higher profits, but lower welfare. In particular, this happens when call demand is iso-elastic, $q(p) = p^{-\eta}$: the (unconstrained) profit maximizing termination mark-up would equal $m^\pi = -c\eta(n\beta-1)/(2\beta(\eta-1)(n-1)) < -p^M/2$, so that Bill and Keep is optimal if negative termination charges are ruled out and $c_T \leq c/2$. Hence, if regulators set $\overline{m}^{\text{RPP}}$ as an upper level on the termination mark-up, firms voluntarily agree on a Bill and Keep regime. This would lead to even larger efficiency losses because lower termination charges (that is, stronger negative termination mark-ups) lead to higher usage prices and thus to lower usage.

4.3 Europe vs. US: comparing CPP and RPP*

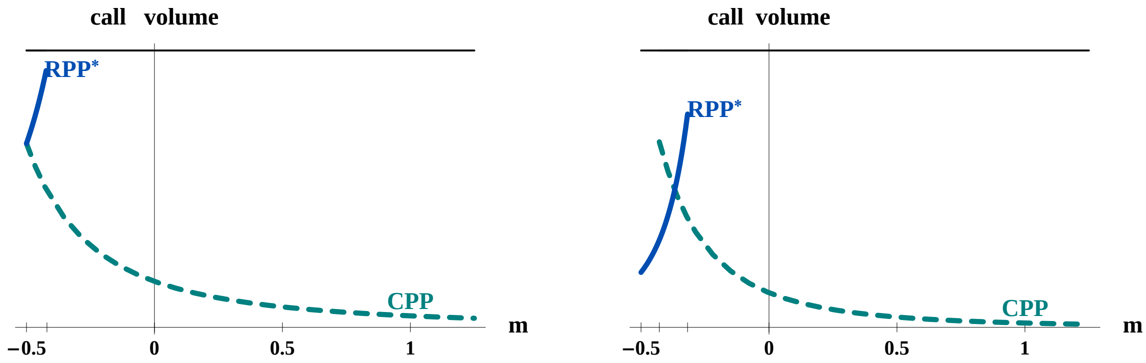
We start illustrating some of the issues at hand by means of two numerical examples. We assume $n = 4$ and $c_O = c_T = 0.5$. In case (a) we further assume that $q(p) = p^{-2}$ and $\beta = 0.8$, while in case (*b*) we assume that $q(p) = p^{-3}$ and $\beta = 0.6$. Figure 3 shows the equilibrium profit as a function of termination mark-up m for the CPP and RPP* equilibria. Note that in case (*a*) the CPP equilibrium exists for any $m \geq -0.5$ (because $\underline{m}^{\text{CPP}} = -0.52$), while the RPP* equilibrium exists only for $m \leq -0.42$. In case (*b*) the CPP equilibrium exists for $m \geq -0.43$ while the RPP* equilibrium exists for $m \leq -0.32$. One observes that equilibrium profit in the CPP equilibrium decreases as termination mark-up is lowered. At some point firms would prefer to switch to the RPP* equilibrium. As the termination mark-up is lowered all the way to Bill and Keep, profit in the RPP*

[30] In order to have efficient on-net call volume with the same price for placing and receiving calls, one needs to set $p = r = r^* = \beta c/(1+\beta)$. The fixed fee must be adjusted accordingly.



(a) $q(p) = p^{-2}$ and $\beta = 0.8$ (b) $q(p) = p^{-3}$ and $\beta = 0.6$

Figure 3: Profit for $n = 4$, $c_O = c_T = 0.5$ in CPP and RPP* equilibrium.



(a) $q(p) = p^{-2}$ and $\beta = 0.8$ (b) $q(p) = p^{-3}$ and $\beta = 0.6$

Figure 4: Call volume for $n = 4$, $c_O = c_T = 0.5$ in CPP and RPP* equilibrium.

equilibrium increases. In fact, the profit under Bill and Keep can even be higher than the maximum profit in any CPP equilibrium as is shown in Figure 3(b).

In order to see how efficiency is affected, we plot call volume in Figure 4. The top level indicates the efficient call volume, which cannot be achieved by any termination mark-up for either type of equilibrium. The lower the termination mark-up, the more efficient is the CPP equilibrium and the less efficient is the RPP* equilibrium. However, depending on the level of the termination mark-up either CPP or RPP* may be better. In particular, in case (a) CPP is outperformed by RPP* in terms of total welfare, but in case (b), there are termination mark-ups where CPP outperforms RPP*. Another striking feature is that under the RPP* equilibrium firms would prefer to use Bill and Keep but regulators would actually prefer higher termination rates. That is, firms voluntarily agreeing on Bill and Keep need not be in the interest of social or consumer welfare.

An eye catching comparison is the one between the (approximate) US scenario of Bill and Keep ($m = -c_T$) and RPP* and the (future) situation in Europe with termination regulated at cost ($m = 0$) and CPP. Let us assume again that $c_O = c_T = c/2$. We compare both scenarios in terms of producer, consumer and total surplus, assuming the same number of operators in both scenarios.

In the US scenario, off-net call and reception price are then equal to $\hat{p}^{\text{US}} = \hat{r}^{\text{US}} = \beta(n-1)c/(2(n\beta-1))$, so that call volume equals $q^{\text{US}} = q(\hat{r}^{\text{US}}/\beta)$. Under the European scenario off-net call price equals $\hat{p}^{\text{EUR}} = (n-1)c/(n-1-\beta)$ and off-net call volume is equal to $q^{\text{EUR}} = q(\hat{p}^{\text{EUR}})$. Call volume would thus be higher (and more efficient) in the European scenario if and only if $\beta < (n+1)/(2n+1)$. On the other hand, the margin earned on each minute of call in the European scenario (i.e., $\hat{p}^{\text{EUR}} - c$) will be higher than the one in the US scenario (i.e., $2\hat{p}^{\text{US}} - c$) if and only if $\beta > \bar{\beta} = (\sqrt{5}-1)/2$. Because $(n+1)/(2n+1) \leq 4/7 < \bar{\beta}$ for all $n \geq 3$, it follows that when the two scenarios are equally efficient, the European producers earn less profits than their US counterparts. This then also implies that consumer surplus must be strictly higher under the European scenario when both scenarios are equally efficient (in terms of total surplus). Another consequence is that for no value of β it can be true that both producer and consumer surplus are the same in both scenarios.

From the deliberations above we have that for intermediate values of $\beta \in [(n+1)/(2n+1), \bar{\beta}]$, European producers earn less profits. However, both for lower and higher values European operators may earn higher profits than their US counterparts. On the one hand, as β approaches 1 the profit margin for US producers converges to zero, while the one for European producers is bounded away from zero. Hence, if call demand is bounded from above, European producers earn higher profits for high values of β. On the other hand, when β approaches $1/(n-1)$, European producers will earn more if call demand is very elastic, despite the lower margin earned on calls, because of the much higher quantity sold.

We summarize our findings in the following proposition. An illustration of this proposition is given in Figure 5. For the case of call demand with constant elasticity η and call externality β, different regions in the (η, β)-space are identified where, respectively, producer, consumer or total surplus are higher under the European scenario (indicated by the sign '+').

Proposition 5. *Let $n \geq 3$ and call demand be bounded from above and sufficiently elastic over the relevant range of prices. Then there exist cut-off values $1/(n-1) < \beta_1 < \beta_2 = (n+1)/(2n+1) < \beta_3 < \beta_4 < 1$ such that*

[i] Producer surplus is higher under the European scenario than under the US scenario when $\beta < \beta_1$ and when $\beta > \beta_4$.

[ii] Consumer surplus is higher under the European scenario than under the US scenario when $\beta < \beta_3$.

[iii] Total surplus is higher under the European scenario than under the US scenario when $\beta < \beta_2$.

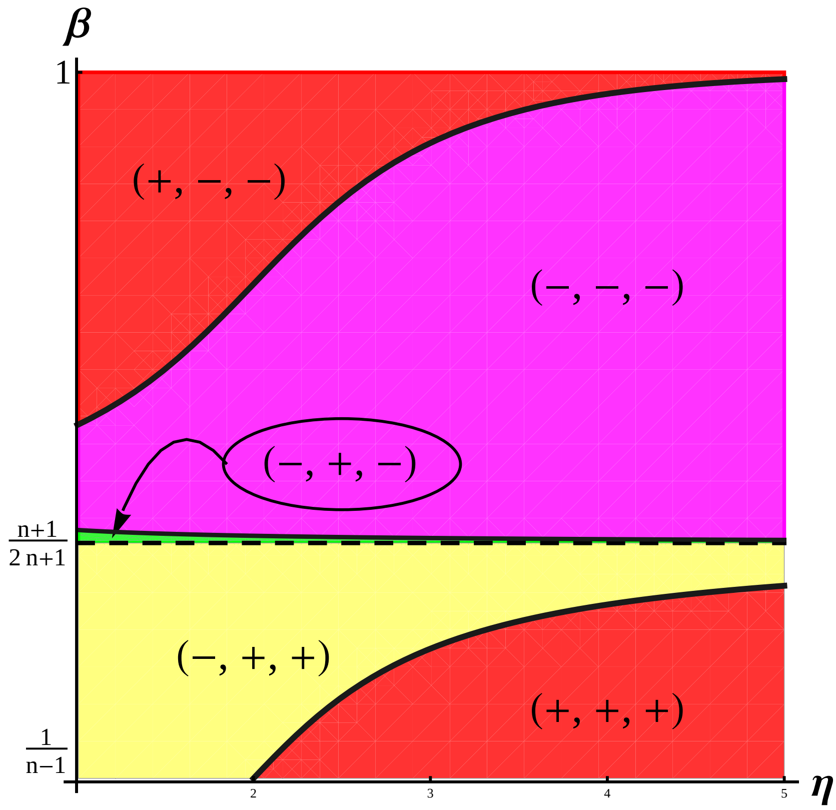


Figure 5: Producer, consumer and total surplus comparison between European and US scenario. (A '+' sign indicates that the relevant welfare measure is higher under the European scenario.)

5 Concluding remarks

In this paper we analyzed the effect of termination charges on competition and welfare in telecommunication markets when receivers obtain utility from incoming calls and network operators can charge call reception. We derived the full set of equilibria (without connectivity breakdown) for any termination charge and analyzed private and social welfare properties.

We showed that efficiency requires that $\hat{p} = p^* = \lambda c$ and $\hat{r} = r^* = (1-\lambda)c$, where $\lambda = 1/(1+\beta)$ denotes the share of the total value of a call enjoyed by the caller. That is, caller and receiver must share the total cost of a call in proportion to the benefit they enjoy from it.[31] This in turn can only occur when the termination charge equals $a^* = c_T - \beta c/(1+\beta)$. Note that with termination charge a^* the perceived marginal cost of originating an off-net call equals $c_O + a^* = \lambda c$ and the perceived marginal cost of terminating an off-net call equals $c_T - a^* = (1-\lambda)c$. Hence, in the efficient equilibrium the total cost of a call must also be shared by the two operators in the same proportion as their subscribers benefit from it. This cost-sharing rule does not depend on how

[31] Degraba (2003) showed that such prices yield efficient consumption, but noted that efficiency also obtains when only one party faces the efficient price while the other party faces a price below the efficient one. We showed that such alternative combination of prices are never part of an equilibrium.

total cost is actually composed. Of course, the actual termination charge does depend on the composition. The originating operator should pay the terminating operator a fraction a^*/c_T of the termination cost. Assuming that $c_O = c_T = c/2$, this fraction equals $a^*/c_T = (1-\beta)/(1+\beta) = 2\lambda - 1$, and decreases from 1 to 0 when β increases from 0 to 1: If caller and receiver share the value of a call equally ($\beta = 1$), Bill and Keep ($a^* = 0$) is optimal. Hence, we obtain an answer to our question of who and how much should be paid for interconnection in the first-best equilibrium.

Despite our result that first-best efficiency is feasible in theory, our analysis suggests that it is unlikely that it will be achieved in practice. Even when termination is fixed at a^*, firms may end up playing a different, and thus necessarily inefficient, equilibrium. There is some hope that firms will be able to overcome this problem because both vanishing noise and coordination on the equilibrium with the highest industry profit select the efficient equilibrium, as we show in Appendix A. However, we also show there that if firms indeed play according one of these equilibrium selection theories, they would have incentives to agree on lower termination charges, increasing profit at the expense of consumer and total welfare. The regulator should thus not allow firms to come to mutual agreements but rather impose both a ceiling and a floor to termination rates.

If firms do not play according to these equilibrium selection theories but play the CPP equilibrium (as they seem to do in Europe) or the RPP* equilibrium (as they seem to do in the US), full efficiency is impossible. Assuming again that $c_T = c/2$, constrained efficiency (that is, maximal efficiency given the type of equilibrium played) requires a termination rate below a^* with CPP but above a^* with RPP*. This suggests that termination rates in Europe should be lowered even below cost and that Bill and Keep regimes in the US should not be overrated from an efficiency point of view. Indeed, the CPP regime in Europe together with cost-based termination (as recommended by the EC) may actually outperform the RPP* regime in the US with voluntary Bill and Keep arrangements in terms of efficiency (and also profitability).

If price discrimination between on- and off-net traffic can be prohibited by the regulator, then all these difficulties can be easily overcome. Hurkens and López (2011) show that in this case a double profit neutrality result holds. Firms make exactly the same profit for any termination rate and in any equilibrium. Coordinating on the efficient rate and efficient equilibrium should then not be too problematic. However, firms make higher profits when price discrimination is allowed and may oppose against such a regulatory intervention that restricts retail pricing.

It would be extremely important to have a better idea of how strong call externalities actually are.[32] This is presumably an empirical matter. We believe that our model can be

[32] Hurkens and López (2012) calibrate welfare gains from regulation in the Spanish telecom market (assuming passive expectations), while Harbord and Hoernig (2015) do so for the UK (assuming responsive expectations). In both cases a CPP regime is assumed and results depend strongly on the strength of the call externality. Rojas (2017) estimates a call externality of about $\beta = 2/3$ based on non-incentivized

used to empirically estimate the strength of the call externality and give reliable results. This would then be useful for the discussion about the pros and cons of RPP and CPP regimes and for future policy about the regulation of termination rates.

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Appendix A Other equilibrium selection

For completeness, we now briefly discuss two further equilibrium selection theories. First we discuss noisy receiver utility, that has been used in several related papers before. Finally, we discuss the idea that firms coordinate on the most profitable equilibrium.

A.1 Noisy receiver utility

Here we follow the approach first introduced in JLT and later also applied by Cambini and Valletti (2008), López (2011), and Hoernig (2016). The idea is that the marginal utility a receiver obtains from a call is subject to some noise, so that, whatever the call and reception charges are, there is a positive probability that the caller hangs up first and there is a positive probability that the receiver hangs up first. This implies that call and reception charges jointly determine call volume so that they can be uniquely determined. By considering the limiting case where the noise vanishes (in a regular way) this method selects a unique equilibrium among the set of equilibria in the game without noise. In the selected equilibrium the caller (respectively, the receiver) determines the volume if $m > \bar{m}$ (respectively, if $m < \bar{m}$). Because reception and termination charges are restricted to be non-negative, the next proposition distinguishes among three regions.

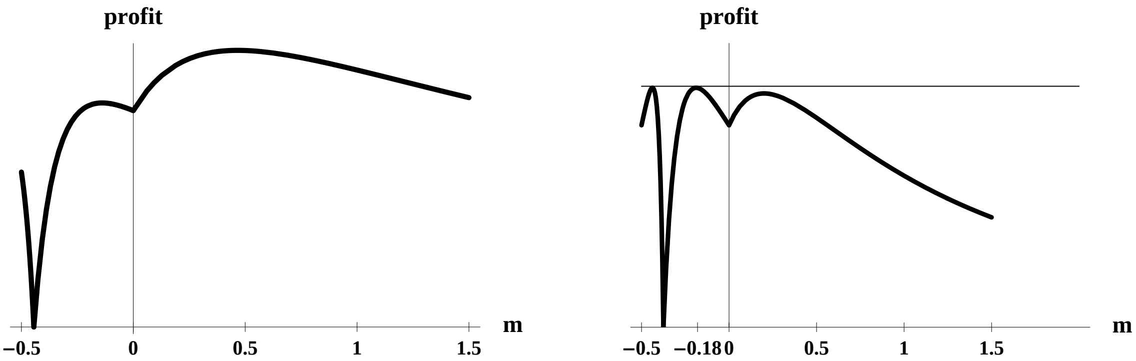
Proposition 6. *The criterion of vanishing noise selects the following equilibrium, depending on the termination mark-up m:*

[i] If $m \geq 0$, the equilibrium selected has $\hat{r} = 0$ and $\hat{p} = (n-1)(c+m)/(n-1-\beta)$. Reception (off-net) is not charged and the caller determines the call volume.

[ii] If $\overline{m} \leq m \leq 0$, the equilibrium selected has $\hat{r} = -m$ and $\hat{p} = ((n-1)c + nm)/(n-1-\beta)$. Reception (off-net) is charged but the caller determines the call volume.

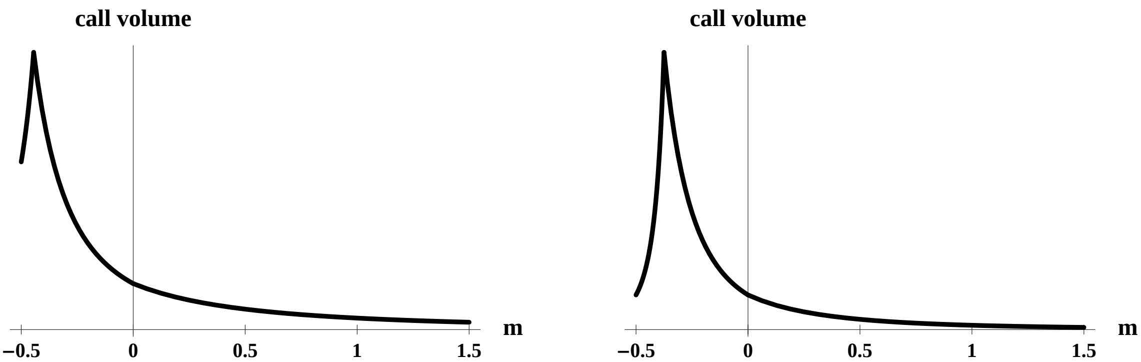
[iii] If $-c_T \leq m \leq \overline{m}$, the equilibrium selected has $\hat{p} = c + m$ and $\hat{r} = -\beta(c + nm)/((n-1)\beta - 1)$. The receiver determines call volume.

It is easily verified that call volume in the selected equilibrium is increasing in m for $m < \overline{m}$ and decreasing in m for $m > \overline{m}$, and that the efficient call volume is obtained when $m^W_{\text{noise}} = \overline{m}$. Note that this socially efficient termination mark-up is not optimal for firm's profits. One can verify that $\hat{p} + \hat{r}$ is increasing in m at $m > \overline{m}$, and that at $m = \overline{m}$, $\hat{p} + \hat{r} = p^* + r^* = c$. From the expression of equilibrium profit in Proposition 1 it follows that increasing m above $\overline{m}$ will increase each firm's profit. It is not straightforward to determine the profit-maximizing termination mark-up for general call demand functions. Depending on the elasticity of call demand and the strength of the call externality, the optimal termination mark-up for firms can be strictly positive (leading to a CPP outcome) or can be negative (leading to an RPP outcome). For example, consider a case with $n = 4$ firms and $c_O = c_T = 0.5$. When $q(p) = p^{-2}$ and $\beta = 0.8$, the profit maximizing termination mark-up is about 0.5. (See Figure 6(a).) When instead $q(p) = p^{-3}$ and



(a) $q(p) = p^{-2}$ and $\beta = 0.8$ (b) $q(p) = p^{-3}$ and $\beta = 0.6$

Figure 6: Profit for $n = 4$, $c_O = c_T = 0.5$ in equilibrium selected by vanishing noise.



(a) $q(p) = p^{-2}$ and $\beta = 0.8$ (b) $q(p) = p^{-3}$ and $\beta = 0.6$

Figure 7: Call volume for $n = 4$, $c_O = c_T = 0.5$ in equilibrium selected by vanishing noise.

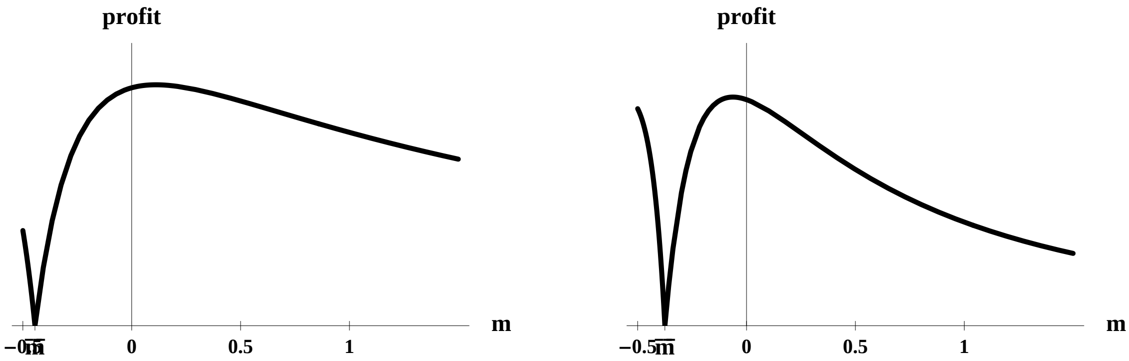
$\beta = 0.6$ the optimal termination mark-up is about $-0.18 > \overline{m} = -0.375$.[33] (See Figure 6(b).) Call volume is always efficient at $\overline{m}$. (See Figures 7(a) and 7(b).)

A.2 Coordination

Let us imagine that for any given termination charge firms coordinate on the equilibrium with the highest (joint) profit. The profit maximizing equilibrium is the one where $\hat{r} = \beta\hat{p}$. This is so because it is the equilibrium with the highest call volume and the largest non-negative profit margin $\hat{p} + \hat{r} - c$. For $m \geq \overline{m}$, the equilibrium has $\hat{r} = \beta\hat{p} = \beta(c+m)$, so that $\hat{p} + \hat{r} - c = (1+\beta)(c+m) - c \geq 0$. For $m \leq \overline{m}$, the equilibrium has $\hat{r} = \beta\hat{p} = -m$, so that $\hat{p} + \hat{r} - c = -m(1+\beta)/\beta - c \geq 0$. (See Figure 1.)

The termination mark-up that maximizes efficiency is the one that yields call volume $q(c/(1+\beta))$. It is straightforward to see that $m^W_{\text{coor}} = \overline{m}$. The profit maximizing termination mark-up above $\overline{m}$ is the one that maximizes $((1+\beta)(c+m) - c)q(c+m)$. (This will

[33]In this case a termination mark-up of $m = -0.44$ also maximizes firm's profit. This is due to the duality between equilibria with caller and receiver determined volume explained in Proposition 4.



(a) $q(p) = p^{-2}$ and $\beta = 0.8$

(b) $q(p) = p^{-3}$ and $\beta = 0.6$

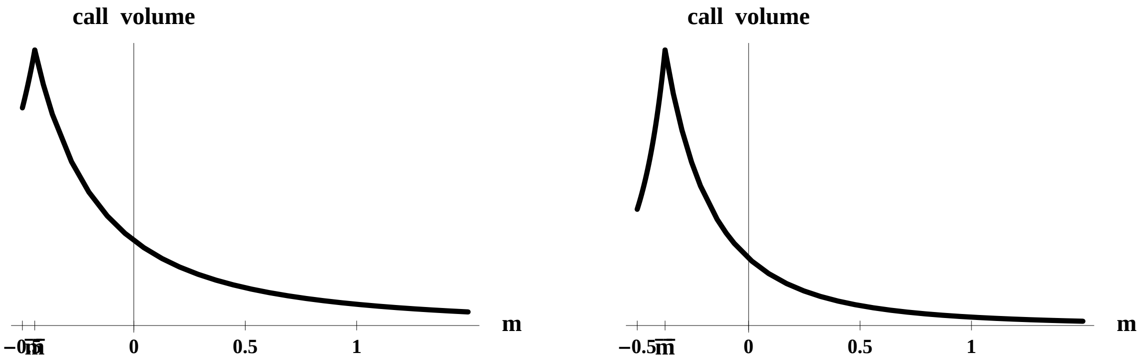
Figure 8: Profit for $n = 4$, $c_O = c_T = 0.5$ in equilibrium selected by coordination.

yield the maximal profit over all equilibria, see Proposition 3.) In the case of iso-elastic call demand $q(p) = p^{-\eta}$, the optimal termination mark-up is such that total price $(\hat{p} + \hat{r})$ equals monopoly price p^M. Call volume will be larger than under monopoly, because it is given by $q(p^M/(1+\beta))$. To be precise, $m^\pi = p^M/(1+\beta) - c$. Because of duality, the same optimal profit could, in principle, also be obtained with a termination mark-up below $\overline{m}$, namely with $m^W = -\beta p^M/(1+\beta)$.[34] However, this would often require a negative termination charge. In case (a), the profit-maximizing termination mark-up is equal to 1/9, in case (b) it is equal to $-1/16$. (See Figure 8). So in the first case, if firms could decide on termination mark-up and equilibrium, they would choose a positive mark-up but still use an RPP business model. If regulators would try to impose $\overline{m}$ as an upper level on the termination mark-up, firms would voluntarily agree on Bill and Keep. As Figure 9 shows, such a jump to Bill and Keep would reduce efficiency compared to the first best situation, especially in case (b). Bill and Keep is only somewhat better than cost-based termination charges. More generally, Bill and Keep yields call volume $q(c_T/\beta)$ while a cost-based termination charge yields call volume $q(c)$. Hence, Bill and Keep is more efficient than cost-based termination when firms coordinate on the profit maximizing equilibrium if and only if $\beta > c_T/c$.

Appendix B The role of expectations

We have assumed throughout that consumers form passive expectations about network sizes. That is, consumers form expectations before firms set prices and do not change them once they observe the prices set by all firms. This is in contrast to the previous literature on termination (with the exception of Hurkens and López, 2014) that assumes consumers form responsive expectations, so that expectations are perfectly adjusted when prices are changed. The type of expectations does not affect the equilibrium off-net call

[34] This would again yield total price equal to monopoly price and call volume as before.



(a) $q(p) = p^{-2}$ and $\beta = 0.8$

(b) $q(p) = p^{-3}$ and $\beta = 0.6$

Figure 9: Call volume for $n = 4$, $c_O = c_T = 0.5$ in equilibrium selected by coordination.

and reception prices. The strategic marginal cost pricing principle holds with responsive and passive expectations because in both cases one uses the standard trick to maximize profit holding market shares constant at $1/n$ (so that responsive and passive expectations coincide). However, the type of expectations is crucial for determining fixed fees and profits. When off-net traffic provides less net consumer surplus than on-net traffic (per call) then consumers prefer to belong to the larger network. Under the assumption of responsive expectations consumers realize that a firm that unilaterally lowers its fixed fee will become the larger network. Hence, some consumers switch to that network not just because of the lower fixed fee but also because they foresee that they will place and receive more on-net calls in comparison to other networks. Under responsive expectations competition is thus fiercer than under passive expectations whenever off-net traffic provides less surplus to consumers than on-net traffic.

This was already pointed out by Hurkens and López (2014) in case reception is exogenously determined to be free of charge. It must thus also hold here when firms play the CPP equilibrium. For example, if firms play the equilibrium selected by vanishing noise and $m > 0$, then fixed fees and profits are lower under responsive expectations than under passive expectations. Even stronger, equilibrium profit increases under passive expectations and decreases under responsive expectations when m is raised above 0. Given the observed opposition by firms against lowering termination charges in European countries, the predictions obtained under passive expectations seem more plausible than those obtained under responsive expectations.

In order to understand better the role of expectations when reception is charged, we calculate fixed fees and equilibrium profit for the case of responsive expectations for the equilibrium robust to vanishing noise.[35]

[35] JLT focus on efficiency and connectivity breakdown but do not calculate fixed fees or profits. Cambini and Valletti (2008) do examine profit but restrict attention to equilibria in which the caller determines volume (most of the time). They show that as the termination charge is lowered towards the efficient level, equilibrium profits go up. Because for termination charges below the efficient level no equilibrium exists in which the caller determines volume and no connectivity breakdown occurs, they conclude that

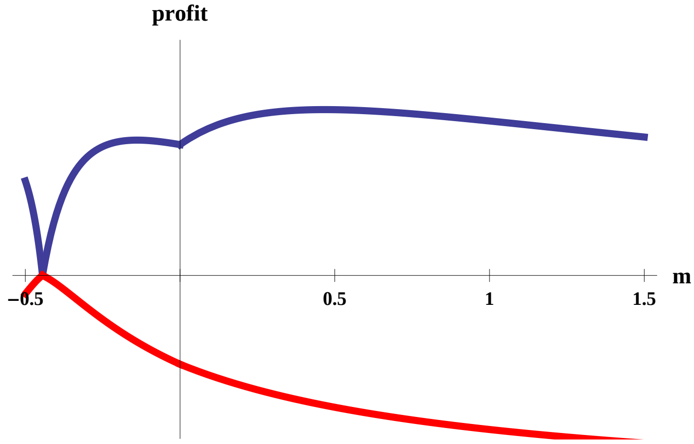


Figure 10: Profit for $n = 4$, $c_O = c_T = 0.5$, $q(p) = p^{-2}$ and $\beta = 0.8$ in equilibrium selected by vanishing noise under **passive** (upper curve) and **responsive** (lower curve) expectations.

Proposition 7. *[i] Assuming responsive expectations and vanishing noise, the symmetric equilibrium is* $(p^*, r^*, \hat{p}, \hat{r}, \tilde{F})$ *where* $\hat{p}$ *and* $\hat{r}$ *are as in Proposition 6. Denoting consumer surplus from on-net calls by*

$$v^* = (1 + \beta)u(q(p^*)) - cq(p^*)$$

and consumer surplus from off-net calls by

$$\hat{v} = (1 + \beta)U(\hat{p}, \hat{r}) - (\hat{p} + \hat{r})D(\hat{p}, \hat{r}),$$

fixed fee equals

$$\widetilde{F} = f + \frac{n\mu}{n-1} - \frac{v^* - \hat{v}}{n-1} - \frac{n-2}{n}(\hat{p} + \hat{r} - c)D(\hat{p}, \hat{r})$$

and equilibrium profit equals

$$\widetilde{\pi} = \frac{\mu}{n-1} - \frac{v^* - \hat{v}}{n(n-1)} + \frac{1}{n^2}(\hat{p} + \hat{r} - c)D(\hat{p}, \hat{r}).$$

[ii] Maximal profit is obtained for $m = \overline{m}$, *so that* $\hat{p} = p^*$ *and* $\hat{r} = r^*$. *That is, free negotiation between firms about a reciprocal termination rate leads to the efficient outcome.*

breakdown must occur for inefficiently low termination charges. They then verify that an equilibrium with connectivity breakdown yields lower profit than the equilibrium without breakdown and thus conclude that free commercial negotiations lead firms to set efficient charges. In contrast, we will allow that when termination charge is set below the efficient level, firms play the equilibrium in which receivers determine volume and still show that under the assumption of responsive expectations firms will voluntarily agree on the efficient termination charge. Our work was done prior to Hoernig (2016) who obtains the same result.

One sees immediately that the difference between equilibrium profits under passive and responsive expectations is captured by the term $(v^* - \hat{v})/(n(n-1))$. Raising off-net surplus increases fixed fee and profit. It should thus not come as a surprise that firms prefer to agree on the efficient termination rate under responsive expectations. Figure 10 compares profits under passive and responsive expectations for the equilibrium selected by vanishing noise.[36] Observe that the profits under passive and responsive expectations agree exactly when the efficient termination charge is set. This is so because then off-net prices are efficient and thus equal to the on-net prices. This implies that then the tariff-mediated network effect disappears and the type of expectations is thus irrelevant. As the termination charge is moved away from the efficient one, the profit under responsive expectations decreases while under passive expectations it increases.

Appendix C Proofs

Proof of Proposition 1.

We start by fixing $\hat{r}_i = \hat{r}^*$ for all $i \in N$. Since subscription demand is assumed inelastic and the off-net call price $\hat{p}_i$ will affect all rivals in the same way (in a symmetric equilibrium), one can calculate the optimal off-net call price of network i by keeping market shares constant (by adjusting F_i accordingly). Fixing $\hat{p}_j = \hat{p}^*$ and $F_j = F^*$ for all $j \neq i$, the profit of network i is equal to

$$\pi_i = \alpha_i \Big[(1-\alpha_i)(\hat{p}_i - c - m)q(\hat{p}_i) + (1-\alpha_i)(\hat{r}^* + m)q(\hat{p}^*) + F_i - f\Big],$$

where F_i is such that the expected surplus from subscribing to network i is equal to the expected surplus from subscribing to any other network:

$$\begin{aligned} F_i = {} & \frac{n-1}{n}\Big[(u(q(\hat{p}_i)) - \hat{p}_i q(\hat{p}_i)) - (u(q(\hat{p}^*)) - \hat{p}^* q(\hat{p}^*))\Big] \\ & + \frac{1}{n}\Big[(\beta u(q(\hat{p}^*)) - \hat{r}^* q(\hat{p}^*)) - (\beta u(q(\hat{p}_i)) - \hat{r}^* q(\hat{p}_i))\Big] + F^*. \end{aligned}$$

Observe that

$$\frac{\partial F_i}{\partial \hat{p}_i} = \frac{n-1}{n}[-q(\hat{p}_i)] - \frac{1}{n}[(\beta \hat{p}_i - \hat{r}^*)q'(\hat{p}_i)].$$

The first-order condition reads

$$\begin{aligned} 0 = \frac{\partial \pi_i}{\partial \hat{p}_i} &= \alpha_i \left[(1-\alpha_i)[q(\hat{p}_i) + (\hat{p}_i - c - m)q'(\hat{p}_i)] - \frac{n-1}{n}q(\hat{p}_i) - \frac{1}{n}(\beta \hat{p}_i - \hat{r}^*)q'(\hat{p}_i)\right] \\ &= \alpha_i q'(\hat{p}_i)\left[(1-\alpha_i)(\hat{p}_i - c - m) - \frac{1}{n}(\beta \hat{p}_i - \hat{r}^*)\right] + \alpha_i q(\hat{p}_i)\left(\frac{1}{n} - \alpha_i\right), \qquad (15) \end{aligned}$$

[36] Note that the profit under passive expectations for this case was already drawn in Figure 6(a). Profits have been rescaled to better illustrate the difference between passive and responsive expectations.

so that in a symmetric equilibrium (where $\alpha_i = 1/n$) with $q'(\hat{p}_i) \neq 0$, we must have

$$(n-1-\beta)\hat{p}_i - (n-1)(c+m) + \hat{r}^* = 0.$$

Note that the second-order derivative of profits, evaluated at the solution of the first-order condition, reads

$$\frac{\partial^2 \pi_i}{\partial \hat{p}_i^2} = \frac{q'(\hat{p}_i)}{n^2}(n-1-\beta) < 0$$

for all $\beta < 1$ and $n \geq 2$. Hence, in a symmetric equilibrium in which callers determine the volume of calls, we must have

$$\hat{p}^* = \frac{(n-1)(c+m) - \hat{r}^*}{n-1-\beta}$$

and $0 \leq \hat{r}^* \leq \beta\hat{p}^*$ or, equivalently, $0 \leq \hat{r}^* \leq \beta(c+m)$. Substituting these prices into the profit function yields

$$\pi_i = \alpha_i[(1-\alpha_i)(\hat{p}^* + \hat{r}^* - c)q(\hat{p}^*) + F_i - f].$$

To find the equilibrium fixed fee we solve the first-order condition[37]

$$0 = \frac{\partial \pi_i}{\partial F_i} = -\frac{\alpha_i(1-\alpha_i)}{\mu}((1-2\alpha_i)(\hat{p}^* + \hat{r}^* - c)q(\hat{p}^*) + F_i - f) + \alpha_i.$$

At a symmetric equilibrium $\alpha_i = 1/n$ so that equilibrium fixed fee satisfies

$$F^* = f + \frac{n\mu}{n-1} - \frac{n-2}{n}(\hat{p}^* + \hat{r}^* - c)q(\hat{p}^*), \tag{16}$$

and equilibrium profit equals

$$\pi^* = \frac{\mu}{n-1} + \frac{1}{n^2}(\hat{p}^* + \hat{r}^* - c)q(\hat{p}^*). \tag{17}$$

Next we show that a necessary condition for $(\hat{p}^*, \hat{r}^*)$ as defined in (5) to be off-net usage prices in a symmetric equilibrium in which the caller determines call volume is that $\beta\hat{p}^* \geq -m$. We also show that when $(n-1)\beta > 1$ and the product differentiation parameter μ is sufficiently high, this necessary condition is also sufficient.

Suppose a network considers to raise the reception charge for off-net calls above $\beta\hat{p}^*$. Such a deviation would make the receivers of this network determine the volume of calls received from subscribers from rival networks. All firms $j \neq i$ set $\hat{p}_j = \hat{p}^*$, $\hat{r}_j = \hat{r}^*$, and $F_j = F^*$ where $\hat{p}^*$ and $\hat{r}^*$ satisfy (5) and F^* satisfies (16) while firm i sets $\hat{p}_i = \hat{p}^*$,

[37] We use here that $\partial\alpha_i/\partial F_i = -\alpha_i(1-\alpha_i)/\mu$ from Eq. (1).

$\hat{r}_i > \beta\hat{p}^*$, and F_i. The profit of firm i is then equal to

$$\pi_i = \alpha_i((1-\alpha_i)(\hat{p}^* - c - m)q(\hat{p}^*) + (1-\alpha_i)(\hat{r}_i + m)q(\hat{r}_i/\beta) + F_i - f).$$

As before, when considering an alternative reception charge $\hat{r}_i$ one can keep market share constant at $1/n$ by adjusting F_i accordingly. That is,

$$\begin{aligned} F_i = & \frac{n-1}{n}\left[(\beta u(q(\hat{r}_i/\beta)) - \hat{r}_i q(\hat{r}_i/\beta)) - (\beta u(q(\hat{p}^*)) - \hat{r}^* q(\hat{p}^*))\right] \\ & + \frac{1}{n}\left[(u(q(\hat{p}^*) - \hat{p}^* q(\hat{p}^*)) - (u(q(\hat{r}_i/\beta)) - \hat{p}^* q(\hat{r}_i/\beta))\right] + F^* \end{aligned}$$

Observe that

$$\frac{\partial F_i}{\partial \hat{r}_i} = \frac{n-1}{n}[-q(\hat{r}_i/\beta)] - \frac{1}{n\beta^2}[(\hat{r}_i - \beta\hat{p}^*)q'(\hat{r}_i/\beta)].$$

Keeping market share α_i constant at $1/n$, the first-order derivative of profit w.r.t. $\hat{r}_i$ is

$$\begin{aligned} \partial\pi_i/\partial\hat{r}_i &= \frac{1}{n}\left[\frac{n-1}{n}[q(\hat{r}_i/\beta) + (\hat{r}_i + m)q'(\hat{r}_i/\beta)/\beta - q(\hat{r}_i/\beta)] - \frac{1}{n\beta^2}(\hat{r}_i - \beta\hat{p}^*)q'(\hat{r}_i/\beta)/\beta\right] \\ &= \frac{q'(\hat{r}_i/\beta)}{\beta^2 n^2}\left[\beta(n-1)(\hat{r}_i + m) - \hat{r}_i + \beta\hat{p}^*\right]. \end{aligned}$$

Note that if $(n-1)\beta - 1 < 0$, the profit function is U-shaped while if $(n-1)\beta - 1 > 0$, the profit function is inversely U-shaped. Moreover, at $\hat{r}_i = \beta\hat{p}^*$

$$\frac{\partial\pi_i}{\partial\hat{r}_i} > 0 \text{ if and only if } \beta\hat{p}^* + m < 0.$$

Hence, if $\beta\hat{p}^* + m < 0$ firm i will certainly want to deviate since even a marginal deviation above $\beta\hat{p}^*$ would be profitable. On the other hand, if $\beta\hat{p}^* + m > 0$ marginal deviations are not profitable. If moreover, $(n-1)\beta - 1 > 0$, then there is no profitable deviation at all, in which market shares are kept constant. By continuity there is no profitable deviation either in which market share is changed slightly. When μ is sufficiently high provoking larger market share deviations is very costly and cannot be profitable. Finally, if $\beta\hat{p}^* + m > 0$ and $(n-1)\beta - 1 < 0$ deviating to $\hat{r}_i = \infty$ may be profitable. ■

Proof of Proposition 2.

Let us fix $\hat{p}_i = \hat{p}^*$ for all $i \in N$. Since subscription demand is assumed inelastic and the off-net reception price $\hat{r}_i$ will affect all rivals in the same way (in a symmetric equilibrium), one can calculate the optimal off-net reception price of network i by keeping market shares constant by adjusting F_i accordingly. Fixing $\hat{r}_j = \hat{r}^*$ and $F_j = F^*$ for all

$j \neq i$, the profit of network i is equal to

$$\pi_i = \alpha_i \Big[(1-\alpha_i)(\hat{p}^* - c - m) q(\hat{r}^*/\beta) + (1-\alpha_i)(\hat{r}_i + m) q(\hat{r}_i/\beta) + F_i - f \Big],$$

where F_i is such that expected surplus from subscribing to network i is equal to the expected surplus obtained from subscribing to any other network:

$$F_i = \frac{n-1}{n} \Big[(\beta u(q(\hat{r}_i/\beta)) - \hat{r}_i q(\hat{r}_i/\beta)) - (\beta u(q(\hat{r}^*/\beta)) - \hat{r}^* q(\hat{r}^*/\beta)) \Big]$$
$$+ \frac{1}{n} \Big[(u(q(\hat{r}^*/\beta)) - \hat{p}^* q(\hat{r}^*/\beta)) - (u(q(\hat{r}_i/\beta)) - \hat{p}^* q(\hat{r}_i/\beta)) \Big] + F^*.$$

Observe that

$$\frac{\partial F_i}{\partial \hat{r}_i} = \frac{n-1}{n} [-q(\hat{r}_i/\beta)] - \frac{1}{n\beta^2} [(\hat{r}_i - \beta \hat{p}^*) q'(\hat{r}_i/\beta)].$$

At a symmetric equilibrium (with market share α_i kept constant at $1/n$) the derivative of profits w.r.t. $\hat{r}_i$ is

$$\frac{\partial \pi_i}{\partial \hat{r}_i} = \frac{1}{n} \left[\frac{n-1}{n} [q(\hat{r}_i/\beta) + (\hat{r}_i + m) q'(\hat{r}_i/\beta)/\beta - q(\hat{r}_i/\beta)] - \frac{1}{n} (\hat{r}_i/\beta - \hat{p}^*) q'(\hat{r}_i/\beta)/\beta \right]$$
$$= \frac{q'(\hat{r}_i/\beta)}{\beta^2 n^2} \left[(n-1)\beta(\hat{r}_i + m) - \hat{r}_i + \beta \hat{p}^* \right].$$

Hence, if $(n-1)\beta - 1 < 0$ no interior solution exists and the optimal reception charge is either $\beta \hat{p}^*$ or ∞. In particular, for $n = 2$ and $\beta < 1$ no equilibrium exists without connectivity breakdown where only the receiver determines call volume.

If $(n-1)\beta - 1 = 0$ a solution exists only if $\beta \hat{p}^* = -m$ (which requires $m \leq 0$). In this peculiar case any reception charge $\hat{r}^* \geq \beta \hat{p}^*$ satisfies the first-order condition.

If $(n-1)\beta - 1 > 0$ a unique interior solution is given by

$$(\beta(n-1) - 1)\hat{r}_i + \beta(n-1)m + \beta \hat{p}^* = 0.$$

Hence, in this case a symmetric equilibrium in which receivers determine the volume of calls must satisfy $0 \leq \hat{\beta} p^* \leq \hat{r}^*$ and

$$\hat{r}^* = \frac{\beta((n-1)m + \hat{p}^*)}{1 - (n-1)\beta}.$$

Substituting the prices into the profit function and maximizing with respect to the fixed fee yields, as before

$$F^* = f + \frac{n\mu}{n-1} - \frac{n-2}{n} (\hat{p}^* + \hat{r}^* - c) q(\hat{r}^*/\beta), \tag{18}$$

so that equilibrium profit equals

$$\pi^* = \frac{\mu}{n-1} + \frac{1}{n^2}(\hat{p}^* + \hat{r}^* - c)q(\hat{r}^*/\beta). \tag{19}$$

We will now check whether a firm i may have an incentive to raise the off-net call price $\hat{p}_i$ above $\hat{r}^*/\beta$. Such a deviation makes the callers of this network determine the volume of off-net calls. Profit of firm i is then equal to

$$\pi_i = \alpha_i((1-\alpha_i)(\hat{p}_i - c - m)q(\hat{p}_i) + (1-\alpha_i)(\hat{r}^* + m)q(\hat{r}^*/\beta) + F_i - f).$$

Note that condition (10) is not yet quite sufficient, as one needs to check whether a network has an incentive to raise call price above $\hat{r}^*/\beta$. Next we address this question. As before, when considering an alternative reception price, one can keep market share constant by adjusting F_i accordingly. That is

$$\begin{aligned} F_i = {} & \frac{n-1}{n}\left[(u(q(\hat{p}_i)) - \hat{p}_i q(\hat{p}_i)) - (u(q(\hat{r}^*/\beta)) - \hat{p}^* q(\hat{r}^*/\beta))\right] \\ & + \frac{1}{n}\left[(\beta u(q(\hat{r}^*/\beta)) - \hat{r}^* q(\hat{r}^*/\beta)) - (\beta u(q(\hat{p}_i)) - \hat{r}^* q(\hat{p}_i))\right] + F^* \end{aligned}$$

Observe that

$$\frac{\partial F_i}{\partial \hat{p}_i} = \frac{n-1}{n}[-q(\hat{p}_i)] - \frac{1}{n}[(\beta\hat{p}_i - \hat{r}^*)q'(\hat{p}_i)].$$

$$\begin{aligned} 0 = \partial\pi_i/\partial\hat{p}_i &= \alpha_i\left[\frac{n-1}{n}[q(\hat{p}_i) + (\hat{p}_i - c - m)q'(\hat{p}_i) - q(\hat{p}_i)] - \frac{1}{n}(\beta\hat{p}_i - \hat{r}^*)q'(\hat{p}_i)\right] \\ &= \frac{q'(\hat{p}_i)}{n^2}\left[(n-1-\beta)\hat{p}_i - (n-1)(c+m) + \hat{r}^*\right] \end{aligned}$$

so that

$$(n-1-\beta)\hat{p}_i - (n-1)(c+m) + \hat{r}^* = 0.$$

Note that the second-order derivative of profits, evaluated at the solution of the first-order condition, reads

$$\frac{\partial^2\pi}{\partial\hat{p}_i^2} = \frac{q'(\hat{p}_i)}{n^2}(n-1-\beta) < 0$$

for all $\beta < 1$ and $n \geq 2$. A profitable (marginal) deviation above $\hat{r}^*/\beta$ thus exists whenever $\partial\pi/\partial\hat{p}_i > 0$, when evaluated at $\hat{p}_i = \hat{r}^*/\beta$. So a necessary condition for the candidate equilibrium to be an equilibrium is that $\hat{r} \geq \beta(c+m)$. When μ is sufficiently high, it will then also not be profitable to deviate and change market share away from $1/n$. For an equilibrium in which the receiver determines the call volume to exist it must be true that $\hat{r} \geq \beta(c+m)$ for $\hat{r} = \beta(n-1)m/[1-(n-1)\beta)]$ (that is for $\hat{p} = 0$). This implies the condition on m. ■

Proof of Proposition 6.

Here we provide the details of equilibrium selection based on vanishing noise. It follows the approach by JLT (2004), who introduced this in a duopoly model assuming rationally responsive expectations. It turns out that neither the type of expectations nor the number of firms is relevant for which candidate equilibrium is selected.[38]

We assume that the utility that a receiver derives from receiving a call of length q is subject to some noise ε: $\beta u(q) + \varepsilon q$, where ε is distributed with cumulative distribution function $F(\cdot)$ on wide enough support $[\underline{\varepsilon}, \bar{\varepsilon}]$, zero mean, and strictly positive density function $f(\cdot)$. Additionally, ε is identically and independently distributed for each caller-receiver pair. We will assume that noise vanishes in the following regular way (similar to JLT):

Definition 2. *A sequence of distributions $F_n(\varepsilon)$ with zero mean on domain $[\underline{\varepsilon}, \bar{\varepsilon}]$ is called regular if for any continuous function $h(\cdot)$ we have*

$$\lim_{n\to\infty} E[h(\varepsilon)|\varepsilon \geq \varepsilon_0] = h(\varepsilon_0) \text{ for all } \varepsilon_0 \geq 0$$

and

$$\lim_{n\to\infty} E[h(\varepsilon)|\varepsilon \leq \varepsilon_0] = h(\varepsilon_0) \text{ for all } \varepsilon_0 \leq 0.$$

As receivers are allowed to hang up, for a given pair of relevant[39] prices (p_i, r_j) the length of a call from a caller of network i to a receiver of network j is given by $q(\max\{p_i, (r_j - \varepsilon)/\beta\})$. Therefore, the volume of calls from network i to network j is $\alpha_i \alpha_j D(p_i, r_j)$ with

$$D(p_i, r_j) = [1 - F(r_j - \beta p_i)]q(p_i) + \int_{\underline{\varepsilon}}^{r_j - \beta p_i} q\left(\frac{r_j - \varepsilon}{\beta}\right) f(\varepsilon) d\varepsilon.$$

Observe that

$$\frac{\partial D(p_i, r_j)}{\partial p_i} = [1 - F(r_j - \beta p_i)]\, q'(p_i), \tag{20}$$

while

$$\frac{\partial D(p_i, r_j)}{\partial r_j} = (1/\beta) F(r_j - \beta p_i) E\left[q'\left(\frac{r_j - \varepsilon}{\beta}\right) \middle| \varepsilon \leq r_j - \beta p_i \right]. \tag{21}$$

The utility that a consumer from network i obtains from placing calls to network j is $\alpha_j U(p_i, r_j)$ with

$$U(p_i, r_j) = [1 - F(r_j - \beta p_i)] u(q(p_i)) + \int_{\underline{\varepsilon}}^{r_j - \beta p_i} u\left(q\left(\frac{r_j - \varepsilon}{\beta}\right)\right) f(\varepsilon) d\varepsilon.$$

[38] The number of firms is of course important to determine whether the candidate equilibrium is indeed an equilibrium, as shown before.

[39] With relevant prices we mean that if $i = j$, then we consider the on-net prices p_i and r_i, while if $i \neq j$ we mean the off-net prices $\hat{p}_i$ and $\hat{r}_j$.

Notice that

$$\frac{\partial U(p_i,r_j)}{\partial p_i} = p_i \frac{\partial D(p_i,r_j)}{\partial p_i}, \tag{22}$$

while

$$\frac{\partial U(p_i,r_j)}{\partial r_j} = (1/\beta)F(r_j-\beta p_i)E\left[\frac{r_j-\varepsilon}{\beta}q'\left(\frac{r_j-\varepsilon}{\beta}\right)\middle|\, \varepsilon \le r_j-\beta p_i\right]. \tag{23}$$

The utility that a consumer from network j obtains from receiving calls from network i is $\alpha_i\widetilde{U}(p_i,r_j)$ with

$$\begin{aligned}\widetilde{U}(p_i,r_j) &= \int_{r_j-\beta p_i}^{\overline{\varepsilon}} \left[\beta u(q(p_i))+\varepsilon q(p_i)\right] f(\varepsilon)d\varepsilon \\ &\quad + \int_{\underline{\varepsilon}}^{r_j-\beta p_i}\left[\beta u\left(q\left(\frac{r_j-\varepsilon}{\beta}\right)\right)+\varepsilon q\left(\frac{r_j-\varepsilon}{\beta}\right)\right] f(\varepsilon)d\varepsilon.\end{aligned}$$

Notice that

$$\frac{\partial \widetilde{U}(p_i,r_j)}{\partial r_j} = r_j \frac{\partial D(p_i,r_j)}{\partial r_j}, \tag{24}$$

while

$$\frac{\partial \widetilde{U}(p_i,r_j)}{\partial p_i} = [1-F(r_j-\beta p_i)]E\left[(\beta p_i+\varepsilon)q'(p_i)\middle|\, \varepsilon \ge r_j-\beta p_i\right]. \tag{25}$$

We will solve for the symmetric equilibrium (without connectivity breakdown). We start the analysis with the market for on-net calls. It is optimal for network i to maximize the size of the pie for on-net calls. The first-order conditions with respect to p_i reads

$$\frac{\partial[U(p_i,r_i)+\tilde{U}(p_i,r_i)-cD(p_i,r_i)]}{\partial p_i}=0,$$

while the one with respect to r_i reads

$$\frac{\partial[U(p_i,r_i)+\tilde{U}(p_i,r_i)-cD(p_i,r_i)]}{\partial r_i}=0.$$

Using the expressions for partial derivatives derived before, the system of equations is equivalent to

$$\begin{cases} 0 &= E\left[(p_i-c+\beta p_i+\varepsilon)q'(p_i)\middle|\, \varepsilon \ge r_i-\beta p_i\right] \\ \\ 0 &= E\left[(r_i-c+\frac{r_i-\varepsilon}{\beta})q'\left(\frac{r_i-\varepsilon}{\beta}\right)\middle|\, \varepsilon \le r_i-\beta p_i\right]\end{cases}$$

Let $(p_i^{(n)}, r_i^{(n)})$ denote the solution when the noise distribution is $F^{(n)}$. Without loss of generality we may assume that the sequence converges to (p,r) and that $\beta p \ge r$ or that $\beta p \le r$. If $r \le \beta p$, then by the regularity of the sequence $F^{(n)}$ it must hold that

$$\begin{cases} 0 &= (p-c+\beta p)q'(p) \\ 0 &= (r-c+p)q'(p)\end{cases}$$

so that $p = c/(1+\beta)$ and $r = \beta c/(1+\beta)$.
If $r \geq \beta p$, then by the regularity of the sequence $F^{(n)}$ it must hold that

$$\begin{cases} 0 &= (p-c+r)q'(p) \\ 0 &= (r-c+r/\beta)q'(r/\beta) \end{cases}$$

so that again $p = c/(1+\beta)$ and $r = \beta c/(1+\beta)$.

Hence, $p = p^* = c/(1+\beta)$ and $r = r^* = \beta c/(1+\beta)$.

We now solve for the optimal off-net call and reception charges. Let us fix the prices of all firms $j \neq i$: $(p^*, r^*, \hat{p}, \hat{r}, F)$. Because consumers have passive expectations and we are considering symmetric equilibrium, all consumers expect market shares to be equal to $1/n$. Expected surplus to subscribing to network i is thus

$$\begin{aligned} w_i &= \frac{1}{n}\Big(U(p^*, r^*) + \widetilde{U}(p^*, r^*) - cD(p^*, r^*)\Big) - F_i \\ &\quad + \frac{n-1}{n}\Big(U(\hat{p}_i, \hat{r}) - \hat{p}_i D(\hat{p}_i, \hat{r}) + \widetilde{U}(\hat{p}, \hat{r}_i) - \hat{r}_i D(\hat{p}, \hat{r}_i)\Big), \end{aligned}$$

while subscribing to any of the rival networks j yields surplus

$$\begin{aligned} w_j &= \frac{1}{n}\Big(U(p^*, r^*) + \widetilde{U}(p^*, r^*) - cD(p^*, r^*)\Big) - F \\ &\quad + \frac{1}{n}\Big(U(\hat{p}, \hat{r}_i) - \hat{p} D(\hat{p}, \hat{r}_i) + \widetilde{U}(\hat{p}_i, \hat{r}) - \hat{r} D(\hat{p}_i, \hat{r})\Big) \\ &\quad + \frac{n-2}{n}\Big(U(\hat{p}, \hat{r}) - \hat{p} D(\hat{p}, \hat{r}) + \widetilde{U}(\hat{p}, \hat{r}) - \hat{r} D(\hat{p}, \hat{r})\Big). \end{aligned}$$

The surplus in the first line corresponds to on-net calls and fixed fee, the surplus in the second line to off-net calls from and to network i, and the surplus in the last line corresponds to off-net traffic from and to other networks.

In order to keep true market share constant when changing $\hat{p}_i$ and $\hat{r}_i$, network i should adjust fixed fee so that $w_i - w_j$ remains constant, so that

$$\begin{aligned} \frac{\partial F_i}{\partial \hat{p}_i} &= -\frac{n-1}{n} D(\hat{p}_i, \hat{r}) - \frac{1}{n}\left(\frac{\partial \widetilde{U}(\hat{p}_i, \hat{r})}{\partial \hat{p}_i} - \hat{r}\frac{\partial D(\hat{p}_i, \hat{r})}{\partial \hat{p}_i}\right) \\ \frac{\partial F_i}{\partial \hat{r}_i} &= -\frac{n-1}{n} D(\hat{p}, \hat{r}_i) - \frac{1}{n}\left(\frac{\partial U(\hat{p}, \hat{r}_i)}{\partial \hat{r}_i} - \hat{p}\frac{\partial D(\hat{p}, \hat{r}_i)}{\partial \hat{r}_i}\right). \end{aligned}$$

Observe that no profit is made on on-net traffic so that

$$\pi_i = \alpha_i (F_i + (1-\alpha_i)[(\hat{p}_i - (c+m))D(\hat{p}_i \hat{r}) + (\hat{r}_i + m)D(\hat{p}, \hat{r}_i)]).$$

Maximizing profit while holding market share constant through adjusting fixed fee F_i thus yields

$$\begin{cases} 0 = \frac{\partial \pi_i}{\partial \hat{p}_i} &= \alpha_i \left(\frac{\partial F_i}{\partial \hat{p}_i} + (1-\alpha_i) \left[D(\hat{p}_i, \hat{r}) + (\hat{p}_i - (c+m)) \frac{D(\hat{p}_i,\hat{r})}{\partial \hat{p}_i} \right] \right) \\ 0 = \frac{\partial \pi_i}{\partial \hat{r}_i} &= \alpha_i \left(\frac{\partial F_i}{\partial \hat{r}_i} + (1-\alpha_i) \left[D(\hat{p}, \hat{r}_i) + (\hat{r}_i + m) \frac{D(\hat{p},\hat{r}_i)}{\partial \hat{r}_i} \right] \right) \end{cases}$$

Using that at a symmetric equilibrium $\alpha_i = 1/n$ these equations can be rewritten as

$$\begin{cases} 0 &= -\frac{\partial \widetilde{U}(\hat{p}_i,\hat{r})}{\partial \hat{p}_i} + \hat{r}\frac{\partial D(\hat{p}_i,\hat{r})}{\partial \hat{p}_i} + (n-1)(\hat{p}_i - (c+m))\frac{D(\hat{p}_i,\hat{r})}{\partial \hat{p}_i} \\ 0 &= -\frac{\partial U(\hat{p},\hat{r}_i)}{\partial \hat{r}_i} + \hat{p}\frac{\partial D(\hat{p},\hat{r}_i)}{\partial \hat{r}_i} + (n-1)(\hat{r}_i + m)\frac{D(\hat{p},\hat{r}_i)}{\partial \hat{r}_i} \end{cases}$$

Substituting Eqs. (20), (21), (23) and (25) and taking out non-negative factors $[1 - F(\hat{r} - \beta\hat{p}_i)]$ and $F(\hat{r}_i - \beta\hat{p})/\beta$, this can be rewritten as

$$\begin{cases} 0 &= E\left[-\beta\hat{p}_i - \varepsilon + \hat{r} + (n-1)(\hat{p}_i - (c+m)) \middle| \varepsilon \geq \hat{r} - \beta\hat{p}_i\right] q'(\hat{p}_i) \\ 0 &= E\left[\left(-\frac{\hat{r}_i - \varepsilon}{\beta} + \hat{p} + (n-1)(\hat{r}_i + m)\right) q'\left(\frac{\hat{r}_i - \varepsilon}{\beta}\right) \middle| \varepsilon \leq \hat{r}_i - \beta\hat{p}\right] \end{cases}$$

Let $F^{(n)}$ represent a series of noise distributions that is regular and let $(\hat{p}^{(n)}, \hat{r}^{(n)})$ denote the corresponding symmetric equilibrium candidate usage prices. By taking a suitable subsequence one may assume that either $\hat{r}^n - \beta\hat{p}^n \leq 0$ for all n or that $\hat{r}^n - \beta\hat{p}^n \geq 0$ for all n.

Consider the first case. Then in the limit, as noise vanishes, the limit point $(\hat{p}, \hat{r})$ must satisfy $\hat{r} - \beta\hat{p} \leq 0$ and

$$\begin{aligned} 0 &= [(n-1-\beta)\hat{p} - (n-1)(c+m) + \hat{r}]q'(\hat{p}_i) \\ 0 &= (n-1)(\hat{r}+m)q'(\hat{p}) \end{aligned}$$

so that (if $m \leq 0$) $\hat{r} = -m$ and $\hat{p} = ((n-1)c + nm)/(n-1-\beta)$. The condition $\hat{r} - \beta\hat{p} \leq 0$ is satisfied if and only if $m \geq \bar{m}$.

Consider the second case next. Then in the limit, as noise vanishes, the limit point $(\hat{p}, \hat{r})$ must satisfy $\hat{r} - \beta\hat{p} \geq 0$ and

$$\begin{aligned} 0 &= (n-1)(\hat{p} - (c+m)) \\ 0 &= (\hat{r}(n-1-1/\beta) + \hat{p} + (n-1)m)q'(\hat{r}/\beta) \end{aligned}$$

so that $\hat{p} = c + m$ and $\hat{r} = -\beta(c+nm)/((n-1)\beta - 1)$. The condition $\hat{r} - \beta\hat{p} \geq 0$ is satisfied if and only if $m \leq \bar{m}$. ∎

Proof of Proposition 7.

[i] The result on call and reception charges follows exactly the analysis under passive expectations. With noise both first-order conditions w.r.t. call and reception charges

(while keeping market share constant by adjusting the fixed fee) are necessary. The limiting case of vanishing noise yields thus exactly the same variable prices as under passive expectations in Proposition 6. In order to determine the fixed fee, we need to take into account that market shares respond differently to changes in fixed fees under responsive expectations. In order to emphasize the fact that we are now assuming responsive expectations, we will use $\widetilde{\alpha}_i$ to denote market share. Profit of firm i is, as before,

$$\widetilde{\pi}_i = \widetilde{\alpha}_i(F_i - f + (1-\widetilde{\alpha}_i)(\hat{p}+\hat{r}-c)D(\hat{p},\hat{r})).$$

Under responsive expectations the change in market share after a change in fixed fee is different: $\partial\widetilde{\alpha}_i/\partial F_i \neq \partial\alpha_i/\partial F_i$. Subscribing to network j yields surplus

$$w_j = \widetilde{\alpha}_j v^* + (1-\widetilde{\alpha}_j)\hat{v} - F_j.$$

Hence, for $j \neq i$

$$\frac{\partial w_j}{\partial F_i} = \frac{\partial \widetilde{\alpha}_j}{\partial F_i}(v^* - \hat{v})$$

and

$$\frac{\partial w_i}{\partial F_i} = \frac{\partial \widetilde{\alpha}_i}{\partial F_i}(v^* - \hat{v}) - 1.$$

Note that

$$\frac{\partial \widetilde{\alpha}_j}{\partial F_i} = -\frac{1}{n-1}\frac{\partial \widetilde{\alpha}_i}{\partial F_i}.$$

Using

$$\frac{\partial \widetilde{\alpha}_i}{\partial F_i} = \frac{\widetilde{\alpha}_i(1-\widetilde{\alpha}_i)}{\mu}\left(\frac{\partial w_i}{\partial F_i} - \frac{\partial w_j}{\partial F_i}\right)$$

we obtain

$$\frac{\partial \widetilde{\alpha}_i}{\partial F_i} = \frac{-\widetilde{\alpha}_i(1-\widetilde{\alpha}_i)}{\mu - \widetilde{\alpha}_i(1-\widetilde{\alpha}_i)(v^*-\hat{v})(n/(n-1))}.$$

At the symmetric equilibrium $\widetilde{\alpha}_i = 1/n$ so that

$$\frac{\partial \widetilde{\alpha}_i}{\partial F_i} = \frac{-(n-1)}{\mu n^2 - n(v^* - \hat{v})}.$$

From the first-order condition $0 = \partial\widetilde{\pi}/\partial F_i$ we obtain

$$\tilde{F} = \frac{-1/n}{\partial\widetilde{\alpha}_i/\partial F_i} + f - \frac{n-2}{n}(\hat{p}+\hat{r}-c)D(\hat{p},\hat{r}).$$

Hence,

$$\tilde{F} = f + \frac{n\mu}{n-1} - \frac{v^*-\hat{v}}{n-1} - \frac{n-2}{n}(\hat{p}+\hat{r}-c)D(\hat{p},\hat{r})$$

and

$$\tilde{\pi}^* = \frac{\mu}{n-1} - \frac{v^*-\hat{v}}{n(n-1)} + \frac{1}{n^2}(\hat{p}+\hat{r}-c)D(\hat{p},\hat{r}).$$

Substituting the equilibrium off-net call and reception charges yields

$$
\begin{aligned}
n^2 \frac{\partial \widetilde{\pi}}{\partial m} &= \frac{n}{n-1} \frac{\partial \hat{v}}{\partial m} + \frac{\partial}{\partial m}[(\hat{p}+\hat{r}-c)D(\hat{p},\hat{r})] \\
&= \frac{n}{n-1}\left[(1+\beta)u' \frac{\partial D}{\partial m} - \frac{\partial(\hat{p}+\hat{r})}{\partial m} D - (\hat{p}+\hat{r})\frac{\partial D}{\partial m}\right] \\
&\quad + \frac{\partial(\hat{p}+\hat{r})}{\partial m} D + (\hat{p}+\hat{r}-c)\frac{\partial D}{\partial m}
\end{aligned}
$$

Note that at $m = \bar{m}$, $\hat{p} = \hat{r}/\beta = c/(1+\beta)$, so that $\hat{p}+\hat{r} = c$ and $u' = c/(1+\beta)$ (both when taking the derivative from the right and from the left). Hence,

$$
n^2 \left. \frac{\partial \widetilde{\pi}}{\partial m} \right|_{m=\bar{m}} = \frac{-D}{n-1} \frac{\partial[\hat{p}+\hat{r}]}{\partial m},
$$

which is negative (positive) when taking derivative from the right (left), as $\hat{p}+\hat{r}$ obtains the minimum at $m = \bar{m}$. This proves that a (local) maximum is obtained at $\bar{m}$. ■